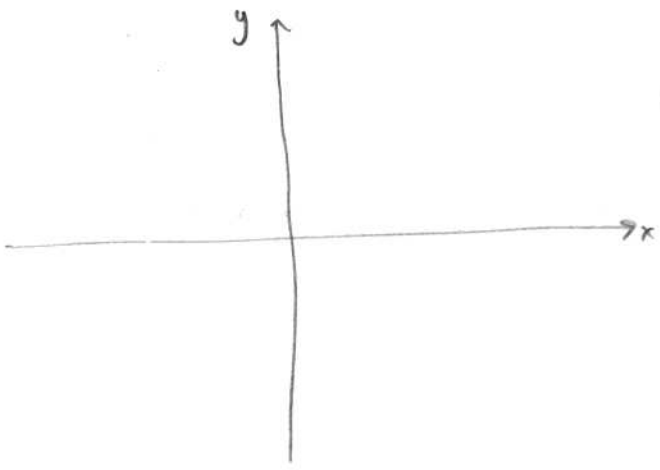
MATH 23 WORKSHEET : Exact equations 10/3/07

Consider $4x^3 + y + (x+2y)y' = 0$

i) Is this an exact equation, ie does a Ψ exist? [Check $M_y = N_x$]

ii) Find Ψ, so write a general soln. to ODE:

iii) BONUS: sketch contour lines of Ψ, hence the direction field.



MATH 23 WORKSHEET : Exact equations 10/3/07

SOLUTIONS

Consider $4x^3 + y + (x+2y)y' = 0$

i) Is this an exact equation, ie does a Ψ exist? [Check $M_y = N_x$]

$M_y = \frac{\partial}{\partial y}(4x^3 + y) = 1$

$N_x = \frac{\partial}{\partial x}(x + 2y) = 1$

} equal → yes, Ψ exists.

ii) Find Ψ, so write a general soln. to ODE:

$\Psi = \int M\,dx + h(y) = x^4 + xy + h(y)$ (careful!)

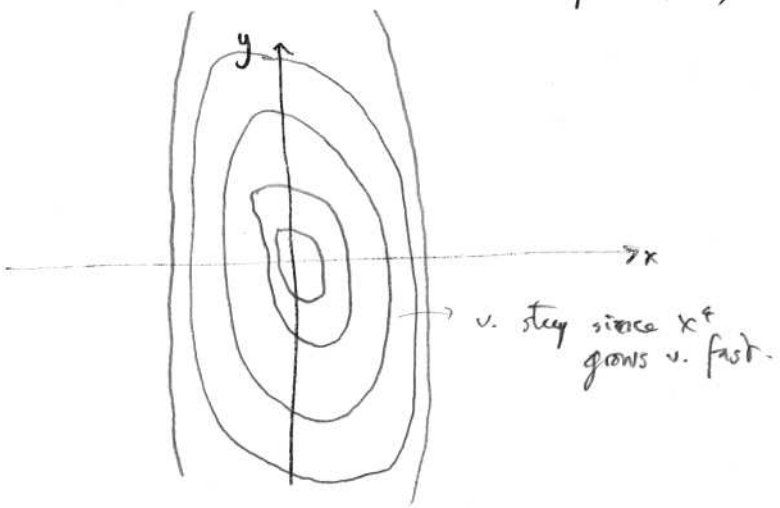
$\frac{\partial \Psi}{\partial y} = \frac{\partial}{\partial y}(x^4 + xy + h(y)) = 0 + x + h'(y) = N = x + 2y$

so $h'(y) = 2y$, → $h(y) = y^2$

⇒ $\Psi(x,y) = x^4 + xy + y^2$

$\boxed{x^4 + xy + y^2 = c}$ gen. soln.

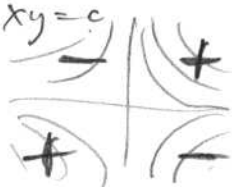
iii) BONUS: sketch contour lines of Ψ, hence the direction field.



→ v. steep since x^4 grows v. fast.

cf. $x^2 + y^2 = c$

$xy = c$



$x^4 + y^2 = c$