Barnett
12/2/03

MATH23 REVIEW.

<u>What method</u> is needed (or best) to solve the following?

(don't actually solve, until chosen all the methods).

a) $y'' + 4y + 3 = t$

b) $y' + ty = t \quad , \quad y(0) = 1$

c) $x' = 2y \quad , \quad y' = -x + y$

d) $y'' + y = \dfrac{1}{1-t}$ for $t < 1$, $y(0) = 1$, $y'(0) = 0$

e) $y'' + y = t \quad , \quad y(0) = 0 \quad , \quad y(\pi) = 0$

f) $y' = -\dfrac{xy^2 + y}{x^2 y + x}$

g) $y_{xx} = -y_{zz}, \quad y(x,0) = y(x,\pi) = 0 \quad y(0,z) = 0 \quad y(1,z) = \cancel{x(\pi - x)}\ z(\pi - z$

h) $y' = \dfrac{-y}{x}$

i) $\dfrac{d^2u}{dx^2} + u = (e^{-x} \sin x)(1 + x)$

j) $u_t = u_{xx}$

k) $\left(\dfrac{d^2}{dx^2} + 1\right) y + \sin x = 0$

MATH23 REVIEW. — SOLUTIONS ~ Barnett 12/2/08

What method is needed (or best) to solve the following?

(don't actually solve, until chosen all the methods).

2nd deriv 0th deriv.

a) $y'' + 4y + \underbrace{3 = t}_{\text{all is part of } g}$ ie rearrange! $\underbrace{y'' + 4y}_{\text{const. coeff } r=\pm 2i} = \underbrace{t-3}_{\text{polynomial}}$

Meth. Und. Coeffs. w/ $Y = At + B$

b) $y' + ty = t$, $y(0) = 1$ ← 1st order linear: $y = \frac{1}{\mu}\left[\int \mu g \, dt + c\right]$. $\mu = e^{t^2/2}$

c) $x' = 2y$, $y' = -x + y$ $\vec{x}' = \begin{pmatrix} 0 & 2 \\ -1 & 1 \end{pmatrix}\vec{x}$ → find eigvecs, eigvals.

d) $\underbrace{y'' + y}_{\text{std. const. coeff. 2nd order lin.}} = \frac{1}{1-t}$ for $t<1$, $y(0) = 1$, $y'(0) = 0$

← not in M. Und. Coeffs. form ⇒ need Var. of Params.

e) $\underbrace{y'' + y}_{r^2=-1 \text{ so } r=\pm i} = t$, $y(0) = 0$, $y(\pi) = 0$

1st order poly ⇒ $Y = At + B$. M. Und. Coeff.

f) $y' = -\frac{xy^2 + y}{x^2y + x}$ use M, N, check if exact ⇒ implicit eqn. ⇒ explicit.

g) $y_{xx} = -y_{zz}$, $y(x,0) = y(x,\pi) = 0$ $y(0,z) = 0$ $y(1,z) = z(\pi - z)$

note: will get sine series in $X(x)$ fncs.

← PDE: $y_{xx} + y_{zz} = 0$ (Laplace's Eqn) Sep. of Var.

h) $y' = -\frac{y}{x}$ separable 1st order. $\frac{y'}{y} = -\frac{1}{x}$ int. → $\ln y = -\ln x + c$

$y = \frac{C}{x}$

i) $\underbrace{\frac{d^2u}{dx^2} + u}_{r=\pm i} = \underbrace{(e^{-x}\sin x)}_{\text{osc}*\text{decay}}\underbrace{(1+x)}_{\text{poly}}$ } is actually in M. Und. Coeffs form.

j) $u_t = u_{xx}$ PDE, Heat eqn: sep. of var.

$Y = e^{-x}\sin x\,(A + Bx) + e^{-x}\cos x\,(C + Dx)$

Yuk!

k) $\left(\frac{d^2}{dx^2} + 1\right) y + \sin x = 0$

a differential operator.

M. Und. Coeffs.

Ie, $\underbrace{y'' + y}_{r=\pm i} = -\sin x$ on-resonant driving.

⇒ $Y = x(A\sin x + B\cos x)$