Answers to the first hour exam

1 (i) Let $A = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, B = \begin{pmatrix} 1 & 2 \end{pmatrix}$.

(ii) Let $A = \begin{pmatrix} 1 & 2 \end{pmatrix}, B = A$.

(iii) Let A be $m \times n$, B be $p \times q$. Since $A + B$ is defined, $m = p$ and $n = q$. Since AB is defined, $p = n$. So both A and B are $n \times n$, so BA is defined.

(iv) $-I_n$ is invertible (its inverse is itself), so A and B are invertible.

2 (i) $\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 2 \\ -4 \\ 6 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 3 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ 6 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ so we can take $S = \{\begin{pmatrix} 2 \\ -4 \\ 6 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}\}$.

(ii) T is not onto since its image is spanned by S, so does not contain all members of R^3.

(iii) No, since it can have three pivots at most, so there will be a free variable.

3 (i) T must be one-to-one. There are three pivots, so there can be no free variable.

(ii) T is not onto. In row-reduced echelon form A will have a row of zeroes and therefore $A\mathbf{x}$ cannot be onto.

4 (i) A is $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$.

(ii) $A^{-1} = \frac{1}{2}\begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$.

(iii) $\begin{pmatrix} x_2 + x_3 \\ x_1 + x_3 \\ x_1 + x_2 \end{pmatrix}$.

5 (i) The standard matrix is $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ -1 & -1 & 0 \end{pmatrix}$. It is not invertible, for the determinant is 0. This may be checked other ways as well.

(ii) By the Invertible Matrix Theorem, T can be neither one-to-one nor onto.