MATH 22 WORKSHEET: Span

6/27/06

Let $\vec{a}_1 = \begin{bmatrix} 0 \\ 2 \\ 5 \end{bmatrix}$ $\vec{a}_2 = \begin{bmatrix} 1 \\ -3 \\ -8 \end{bmatrix}$ $\vec{a}_3 = \begin{bmatrix} -4 \\ 2 \\ h \end{bmatrix}$ h is some number

We want to know: Is $\vec{b} = \begin{bmatrix} 1 \\ 1 \\ k \end{bmatrix}$ in $\text{Span}\{\vec{a}_1, \vec{a}_2, \vec{a}_3\}$?

k is some number

Row reduce the augmented matrix to Echelon Form:

- treat h & k as you would usual numbers

What set of h, k makes linear system

a) consistent & unique?

b) consistent but not unique?

Is $\vec{b}$ in the Span in each case a) & b)? What then is difference between a) & b)?

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Let $\vec{a}_1 = \begin{bmatrix} 0 \\ 2 \\ 5 \end{bmatrix}$ $\vec{a}_2 = \begin{bmatrix} 1 \\ -3 \\ -8 \end{bmatrix}$ $\vec{a}_3 = \begin{bmatrix} -4 \\ 2 \\ h \end{bmatrix}$ h is some number

We want to know: Is $\vec{b} = \begin{bmatrix} 1 \\ 1 \\ k \end{bmatrix}$ in $\text{Span}\{\vec{a}_1, \vec{a}_2, \vec{a}_3\}$? k is some number.

Row reduce the augmented matrix to Echelon Form:

- treat h & k as you would usual numbers
- scale a row to remove fractions. actually, not important

$$\begin{bmatrix} 0 & 1 & -4 & 1 \\ 2 & -3 & 2 & 1 \\ 5 & -8 & h & k \end{bmatrix} \overset{\frac{1}{2}}{\sim} \begin{bmatrix} 1 & -3/2 & 1 & 1/2 \\ 0 & 1 & -4 & 1 \\ 5 & -8 & h & k \end{bmatrix}$$

$R_3 \to R_3 - 5R_1$

$$\sim \begin{bmatrix} 1 & -3/2 & 1 & 1/2 \\ 0 & 1 & -4 & 1 \\ 0 & -1/2 & h-5 & k-5/2 \end{bmatrix}$$

$R_3 \to R_3 + \frac{1}{2}R_2$

$$\sim \begin{bmatrix} 1 & -3/2 & 1 & 1/2 \\ 0 & 1 & -4 & 1 \\ 0 & 0 & h-7 & k-2 \end{bmatrix}$$ Echelon Form.

Examine pivot positions (depending on h, k):

$$\left[\begin{array}{ccc|c} \blacksquare & & & \\ & \blacksquare & & \\ & & \blacksquare & \end{array}\right]$$

$h \neq 7$, any k
consistent, unique

$$\left[\begin{array}{ccc|c} \blacksquare & & & \\ & \blacksquare & & \\ & & & \blacksquare \end{array}\right]$$

free var

ie nonzero.

$h = 7$, $k \neq 2$
inconsistent.

$$\left[\begin{array}{cccc} \blacksquare & & & \\ & \blacksquare & & \\ 0 & 0 & 0 & 0 \end{array}\right]$$

free var

$h = 7$, $k = 2$,
consistent, non unique.

What set of h, k makes linear system

a) consistent & unique? $h \neq 7$, k = anything

b) consistent but not unique? $h = 7$, $k = 2$

} it's all about the pivots

Is $\vec{b}$ in the Span in each case? a) yes, b) yes (both consistent)

But, for a) the weights are unique; in b) they're not.