6/26/17
Barnett

MATH 22 WORKSHEET : Span

A) Is $\vec{b} = \begin{bmatrix} -3 \\ 3 \\ 1 \end{bmatrix}$ in $\text{Span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \right\}$?

[note all vectors are in $\mathbb{R}^3$.]

If so, find the weights in the linear combination that gives $\vec{b}$.

B) Back to $\mathbb{R}^2$ (as in lecture): if $\vec{u}, \vec{v}$ are in $\mathbb{R}^2$, must $\text{Span}\{\vec{u}, \vec{v}\}$ be all of $\mathbb{R}^2$?

Thinking geometrically, name a smaller set that $\text{Span}\{\vec{u}, \vec{v}\}$ could be:

Give two distinct ways this could happen: i)

ii)

Name an even smaller set $\text{Span}\{\vec{u}, \vec{v}\}$ could be:

How must this happen?

BONUS) Given vectors $\vec{u}_1, \dots, \vec{u}_p$ in $\mathbb{R}^m$, $\text{Span}\{\vec{u}_1, \dots, \vec{u}_p\}$ is the set of all linear combinations $\vec{y} = c_1\vec{u}_1 + \dots + c_p\vec{u}_p$, as you'd expect. What types of geometric objects can $\text{Span}\{\vec{u}_1, \dots \vec{u}_p\}$ be? Does it matter if $m<p$, or $m>p$?

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~~~ SOLUTIONS ~~~

A) Is $\vec{b} = \begin{bmatrix} -3 \\ 3 \\ 1 \end{bmatrix}$ in $\text{Span}\left\{ \underbrace{\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}}_{\vec{u}}, \underbrace{\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}}_{\vec{v}} \right\}$?

[note all vectors are in $\mathbb{R}^3$.]

If so, find the weights in the linear combination that gives $\vec{b}$.

Vector eqn is: $c_1 \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 3 \\ 1 \end{bmatrix}$

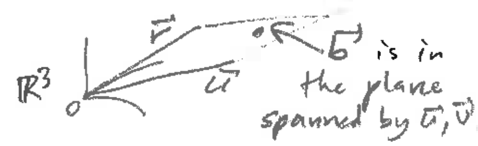
Lin. sys. has aug. matrix $\left[\begin{array}{cc|c} 1 & 2 & -3 \\ 1 & -1 & 3 \\ 3 & 1 & 1 \end{array}\right] \xrightarrow{\text{row reduce}} \left[\begin{array}{cc|c} 1 & 2 & -3 \\ 0 & -3 & 6 \\ 0 & -5 & 10 \end{array}\right]$

EF: 2 pivots:

$\sim \left[\begin{array}{cc|c} 1 & 2 & -3 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{array}\right]$ consistent, can solve by back substitution $c_2 = -2$, $c_1 = -3 - 2c_2 = +1$

or by REF $\sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$

So, yes, $\vec{b}$ is in $\text{Span}\{\vec{u}, \vec{v}\}$



B) Back to $\mathbb{R}^2$ (as in lecture): if $\vec{u}, \vec{v}$ are in $\mathbb{R}^2$, must $\text{Span}\{\vec{u}, \vec{v}\}$ be all of $\mathbb{R}^2$? No.

Thinking geometrically, name a smaller set that $\text{Span}\{\vec{u}, \vec{v}\}$ could be: a line

Give two distinct ways this could happen: i) $\vec{u}$ parallel to $\vec{v}$

ii) $\vec{u}$ anything, $\vec{v} = \vec{0}$ (or vice versa)

Name an even smaller set $\text{Span}\{\vec{u}, \vec{v}\}$ could be: a point, ie $\{\vec{0}\}$.

How must this happen? $\vec{u} = \vec{v} = \vec{0}$ is only way.

BONUS) Given vectors $\vec{u}_1, \cdots, \vec{u}_p$ in $\mathbb{R}^m$, $\text{Span}\{\vec{u}_1, \cdots, \vec{u}_p\}$ is the set of all linear combinations $\vec{y} = c_1\vec{u}_1 + \cdots + c_p\vec{u}_p$, as you'd expect. What types of geometric objects can $\text{Span}\{\vec{u}_1, \cdots \vec{u}_p\}$ be? A point $\{\vec{0}\}$, a line, a plane ... hyperplane dim $\min(m,p)$

Does it matter if $m<p$, or $m>p$? $\min(m,p)$ matters.
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