MATH 22 WORKSHEET : proofs (cont.) & critical thinking. 9/13/16 Barrett

Are the following valid proofs? Only if the claim is true, correct the proof. Discuss so you agree with neighbors:

vi) $ax=b$ always has a solution. Proof: by example, $a=2, b=1$ has solution $x=1/2$. □

vii) If $b=0$, $ax=b$ is consistent. Proof: by contradiction if $b\neq 0$ there's a case $a=0$ which has no solution. So if $b=0$ it must always have one!

viii) $ax=0$ either has one solution or infinitely many.
Proof: There are two cases $a=0$ and $a\neq 0$.
For $a=0$ there are infinitely many (any x is a solution).
For $a\neq 0$ then $x=0$ is the only solution. □

ix) If $ax=b$ has no solution, then $a=0$. Proof: If $a=0$ & $b\neq 0$, it has no solution. □

BONUS : invent convincing but bogus proofs & try out on neighbors.

SOLUTIONS

MATH 22 WORKSHEET : proofs (cont.) & critical thinking. 4/13/16 Barrett

Are the following valid proofs? Only if the claim is true, correct the proof. Discuss so you agree with neighbors:

vi) $ax=b$ always has a solution. Proof: by example, $a=2, b=1$ has solution $x=1/2$. □

False, and not a proof, since would need to check all examples (∞ # of them)

viz: $a=0, b=1$, has no solution.

vii) If $b=0$, $ax=b$ is consistent. Proof: by contradiction if $b\neq 0$ there's a case $a=0$ which has no solution. So if $b=0$ it must always have one □

Claim is True, but proof is bogus: proving something happens for $b\neq 0$ has no bearing on what happens for $b=0$!

A valid proof: $x=0$ is a solution (by example).

viii) $ax=0$ either has one solution or infinitely many.

Proof: there are two cases $a=0$ and $a\neq 0$.
For $a=0$ there are infinitely many (any x is a solution).
For $a\neq 0$ then $x=0$ is the only solution. □

True, & valid proof (enumeration of all cases).

ix) If $ax=b$ has no solution, then $a=0$. Proof: If $a=0$ & $b\neq 0$, it has no solution. □

True, but proof is bogus. It's only proving some converse claim.

A valid proof: by "contrapositive" (look it up): $a\neq 0 \Rightarrow ax=b$ has a solution, namely $x=b/a$. □

BONUS : invent convincing but bogus proofs & try out on neighbors.