

# Lecture 18

Math 22 Summer 2017 July 28, 2017



- §4.6 Rank and the IMT
- §5.1 Eigenvectors and eigenvalues



Let A be a square  $n \times n$  matrix.



- (d) The matrix equation  $A\mathbf{x} = \mathbf{0}$  has only the trivial solution.
- (e) The columns of A form a linearly independent set.
- (g)  $A\mathbf{x} = \mathbf{b}$  has at least one solution for each  $\mathbf{b}$  in  $\mathbb{R}^n$ .
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- (m) The columns of A form a basis of  $\mathbb{R}^n$ .
- (n)  $\operatorname{Col} A = \mathbb{R}^n$ .
- (o) dim  $\operatorname{Col} A = n$ .
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In the situation where  $A\mathbf{x} = \lambda \mathbf{x}$  we say that  $\mathbf{x}$  is an eigenvector corresponding to  $\lambda$ .





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## §5.1 Finding eigenvectors



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This is our first example of an *eigenspace* which we now define...





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What are the eigenspaces for  $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ ?



1. Is 
$$\mathbf{x} = \begin{bmatrix} -6\\5 \end{bmatrix}$$
 an eigenvector for  $A = \begin{bmatrix} 1 & 6\\5 & 2 \end{bmatrix}$ ?  
2. Is  $\lambda = 3$  an eigenvalue of  $A$ ?  
3. Let  $B = \begin{bmatrix} 4 & -1 & 6\\2 & 1 & 6\\2 & -1 & 8 \end{bmatrix}$ . The eigenvalues are  $\lambda = 2, 9$ . Find a basis for the eigenspace corresponding to  $\lambda = 2$ . What is t

basis for the eigenspace corresponding to  $\lambda = 2$ . What is the dimension of this space?

- 4. Using A and x defined above, compute  $A^2 \mathbf{x}, A^3 \mathbf{x}, ..., A^k \mathbf{x}$ .
- Using A defined above, write A λl<sub>2</sub> as a matrix (for arbitrary λ). Now compute det(A λl<sub>2</sub>). For what values of λ is det(A λl<sub>2</sub>) = 0?







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Let  $A = (a_{ij})$ .  $\lambda$  is an eigenvalue of A if and only if the null space of  $A - \lambda I_n$  contains a nonzero vector. Write out  $A - \lambda I_n$  under the assumption that A is triangular to show that  $(A - \lambda I_n)\mathbf{x} = \mathbf{0}$  has a free variable precisely when  $\lambda = a_{kk}$  for some  $k \leq n$ .



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Suppose A has  $\lambda = 0$  as an eigenvalue? What can you say about the invertibility of A?







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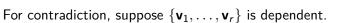
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We will prove this by contradiction.









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 $\mathbf{v}_{p+1}=c_1\mathbf{v}_1+\cdots+c_p\mathbf{v}_p\,.$ 

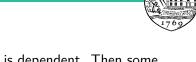


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How can we use the boxed equations to get a contradiction?



# §5.1 Eigenvectors and difference equations





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How can we use this to simplify the computation of  $\mathbf{x}_k$  for large values of k?