Lecture 12

Math 22 Summer 2017
July 14, 2017

Just for today

- Answers to mini-quiz
- Finish §3.2 properties of determinants
- §4.1 abstract vector spaces

§3.2 Examples

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We find that $\det B = \det C = \det A$, $\det D = 2 \det A$, and $\det E = -\det D = -2 \det A$.

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Theorem

A square matrix A is invertible if and only if $\det A \neq 0$.

§3.2 Theorem 5

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If A is an $n \times n$ matrix, the $\det A^T = \det A$.

§3.2 Theorem 6

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If A and B are $n \times n$ matrices, then $\det AB = (\det A)(\det B)$.

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10. $1\mathbf{u} = \mathbf{u}$

§4.1 Basic properties

Try to prove the following properties *using only the axioms*...

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(a) $\mathbf{0}$ is unique

(b) $-\mathbf{u}$ is unique

(c) $0\mathbf{u} = \mathbf{0}$

(d) $c\mathbf{0} = \mathbf{0}$

(e) $-\mathbf{u} = (-1)\mathbf{u}$

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  However, $\mathbb{R}^3$ does have subspaces that are *isomorphic* to $\mathbb{R}^2$.
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- Is any plane in $\mathbb{R}^n$ a subspace?

§4.1 Theorem 1

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Note that we can use all the axioms of the ambient vector space V! □

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$$H = \left\{ \begin{bmatrix} s + 2t \\ -t \\ 3s - 7t \end{bmatrix} : s, t \in \mathbb{R} \right\} \subseteq \mathbb{R}^3.$$

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How can we use the previous theorem to show H is a subspace? Well,

$$H = \text{Span}\left\{ \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ -7 \end{bmatrix} \right\}.$$

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