Lecture 03

Math 22 Summer 2017 Section 2
June 26, 2017

Just for today

- (10 minutes) Review row reduction algorithm
- (40 minutes) §1.3
- (15 minutes) Classwork

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Suppose this is the coefficient matrix of a linear system. What can you say about the solution set?

Recall vectors in $\mathbb{R}^2, \mathbb{R}^3, \mathbb{R}^n$.

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What is the difference between a vector and a scalar?

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What does it mean for two vectors to be equal?

§1.3 Linear combinations

Definition

Given $\mathbf{v}_1, \ldots, \mathbf{v}_p \in \mathbb{R}^n$ and given scalars $c_1, \ldots, c_p \in \mathbb{R}$, we define the **linear combination** of $\mathbf{v}_1, \ldots, \mathbf{v}_p$ with the **weights** $c_1, \ldots, c_p$ by

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- $\mathbf{u} \pm \mathbf{v}, c\mathbf{u} \in \text{Span}\{\mathbf{u}, \mathbf{v}\}$
- $S, T \subseteq \mathbb{R}^n$ and $S \subseteq T$ implies $\text{Span}\{S\} \subseteq \text{Span}\{T\}$

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This means that a vector $\mathbf{b} \in \mathbb{R}^m$ can be expressed as a linear combination of the vectors $\mathbf{a}_1, \ldots, \mathbf{a}_n$ if any only if the linear system corresponding to $\begin{bmatrix}\mathbf{a}_1 & \cdots & \mathbf{a}_n & \mathbf{b}\end{bmatrix}$ is consistent.

§1.3 Classwork

Suppose

$$\begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 & \mathbf{b} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 & -5 \\ -2 & 5 & 0 & 11 \\ 2 & 5 & 8 & -7 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 4/5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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Do there exist scalars $x_1, x_2, x_3 \in \mathbb{R}$ such that

$$x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + x_3\mathbf{a}_3 = \mathbf{b}?$$

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