MATH 22 LECTURE 29 CLASSWORK

AUGUST 23, 2017

(1) Let $A = \begin{bmatrix} -1 & 0 \\ -1 & 0 \end{bmatrix}$.

(a) Maximize $\|A\mathbf{x}\|$ subject to the constraint that $\|\mathbf{x}\| = 1$.

(b) Compute the SVD of A and A^T.

(c) Find orthonormal bases for as many fundamental spaces as possible!

(d) What is the best rank 1 approximation of A?

(2) Let $A = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 2 \\ 0 \\ -5 \end{bmatrix}.$

(a) Find all least-squares solutions to $A\mathbf{x} = \mathbf{b}$.

(b) Let $W = \text{Col}A$. Decompose $\mathbf{b} = \hat{\mathbf{b}} + \mathbf{z}$ with $\hat{\mathbf{b}} \in W$ and $\mathbf{z} \in W^{\perp}$. Prove that this decomposition is unique.

(c) Can we compute the QR-factorization of A?

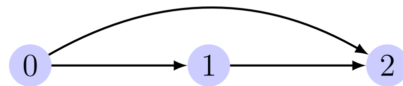
(3) Let $A = \begin{bmatrix} 4 & -2 \\ 1 & 1 \end{bmatrix}$.

(a) Check that A is diagonalizable and diagonalize it.

(b) Is the matrix P unique? If so, prove it. If not, provide an example.

(c) Compute $A^{100} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

(4) Consider the "web" given below:



(a) Use the PageRank algorithm with $\alpha = 1$ to find a probability vector that measures the importance of each node. Explain how the algorithm changes if we instead let $\alpha = 0.85$.

(b) How do you know that the steady-state vector (of the dynamical system defined by A) is unique?