Gram-Schmidt Process and QR-Factorization Worksheet

August 14th, 2017

Let $A = \begin{bmatrix} 3 & -5 & 1 \\ 1 & 1 & 1 \\ -1 & 5 & -2 \\ 3 & -7 & 8 \end{bmatrix}$. Let x_1, x_2, and x_3 be the columns of A. If we let

$$v_1 = x_1 = \begin{bmatrix} 3 \\ 1 \\ -1 \\ 3 \end{bmatrix} \text{ and } v_2 = x_2 - \frac{x_2 \cdot v_1}{v_1 \cdot v_1} v_1 = \begin{bmatrix} -5 \\ 1 \\ 5 \\ -7 \end{bmatrix} - \frac{-40}{20} \begin{bmatrix} 3 \\ 1 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 3 \\ -1 \end{bmatrix}$$

then $\text{Span}\{v_1, v_2\} = \text{Span}\{x_1, x_2\}$ and v_1 and v_2 are orthogonal.

1. Use Gram-Schmidt to find v_3 so that $\{v_1, v_2, v_3\}$ forms an orthogonal basis for Col(A).

2. Normalize v_1, v_2, and v_3. This gives us an orthonormal basis for Col(A).

3. Find the QR factorization of A. The normalized vectors from part 2 make up the columns of Q. One way to find R is to notice that

$$R = Q^T A.$$

   Why is that true? (Hint: last Wednesday's class)