Barnett
7/11/17

→ SOLUTIONS ←

Your name:

Math 22 Summer 2017, mini-quiz 1, Mon July 10

Please show your work. No credit is given for solutions without work or justification.

(1) Write the *parametric vector form* of the solution set to the linear system

3 pts. $x_1 + 2x_2 + 3x_3 = 4$ [Note there is only one equation.]

$A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$ $\vec{b} = \begin{bmatrix} 4 \end{bmatrix}$

Augmented matrix $\begin{bmatrix} 1 & 2 & 3 & | & 4 \end{bmatrix}$

pivot. $\underbrace{x_2 \quad x_3}_{\text{free.}}$

↳ unusual but you've done practise q's in book like this, eg top p.5 already in REF.

General solution. $\begin{cases} x_1 = 4 - 2x_2 - 3x_3 \\ x_2 = x_2 \\ x_3 = x_3 \end{cases}$

or: (vector form). $\vec{x} = \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} s + \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} t$, $s, t \in \mathbb{R}$

parameters. Calling x_2, x_3 is also fine.

(2) Let $\mathbf{v}_1, \ldots, \mathbf{v}_n$ be vectors in $\mathbb{R}^m$. Define $\text{Span}\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$. [Use either mathematical notation or precise mathematical language.]

2 pts.

$\text{Span}\{\vec{v}_1, \ldots, \vec{v}_n\} := \{ c_1\vec{v}_1 + \cdots + c_n\vec{v}_n : c_1, \ldots c_n \text{ are real.} \}$

or in words, the set of all vectors that are linear combinations of the given vectors $\vec{v}_1, \ldots \vec{v}_n$.

(3) Could a set of 7 vectors in $\mathbb{R}^5$ be linearly independent? Prove your answer.

3 pts

No, they could not. Stacking the vectors as columns of a 5×7 matrix A, its echelon form could have a most 5 pivots, hence at least two free variables. Thus the linear system $A\vec{x} = \vec{0}$ has non-trivial solutions (is not unique), which is to say that the vectors are linearly dependent.

(4) Is it possible that a linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^3$ is onto? Prove your answer.

2pts.

Yes. Proof by example: $T(x_1, x_2, x_3, x_4) = (x_1, x_2, x_3)$ has standard matrix $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$,

which is already in E.F., with a pivot in every row, so the linear system $A\vec{x} = \vec{b}$ is consistent for any $\vec{b}$ in $\mathbb{R}^3$, or $T(\vec{x}) = \vec{b}$ has a pre-image $\vec{x}$ for every $\vec{b}$ in $\mathbb{R}^3$, which is the definition of onto.

Alternatively you can state that it is possible for A to have a pivot in every row.