Your name:

Instructor (please circle): Alex Barnett Michael Musty

Math 22 Summer 2017, Homework 6, due Fri August 4 *Please show your work, and check your answers. No credit is given for solutions without work or justification.*

(1) Let $T : \mathbb{R}^5 \to \mathbb{R}^3$ be a linear transformation with standard matrix A.

(a) What are the possible values for the rank of A?

(b) What are the possible values for the dimension of $\text{Nul}\, A$?

(c) Suppose now that the T from above is also onto. What are the possible values for the dimension of $\text{Nul}\, A$?

(d) Let $A = \begin{bmatrix} 2 & 0 & -1 & 2 & 3 \\ 4 & 0 & -2 & 4 & 6 \\ 0 & 0 & 1 & -1 & 0 \end{bmatrix}$. Find bases for $\text{Col}\, A$, $\text{Row}\, A$, and $\text{Nul}\, A$ (In this question you do not need to show your working.)

(2) Let $A = \begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$.

(a) A and B have the same characteristic polynomial. Find this polynomial and explain what form and/or features that A and B have in common make the polynomial the same.

(b) From the previous part we see that each matrix has exactly 2 eigenvalues (call them λ_1 and λ_2 with $\lambda_1 < \lambda_2$). Compute the dimensions of both eigenspaces for both of the matrices A and B. Use this information to fill in the table below. Be sure to include λ_1 and λ_2 in the table. It is not necessary to include *all* the computations involved, but please describe how you computed the dimensions.

matrix	dimension of $\lambda_1 =$ eigenspace	dimension of $\lambda_2 =$ eigenspace
A		
B		

(c) Find a basis for the λ_2 eigenspace of B:

(3) Let $A = \begin{bmatrix} 7 & h \\ 2 & 11 \end{bmatrix}$.

(a) For what value(s) of h does A have an eigenvalue of (algebraic) multiplicity two?

(b) Let λ be the eigenvalue of (algebraic) multiplicity two from the previous part. Compute a basis for the λ eigenspace of A.

(c) Let $P = \begin{bmatrix} -2 & 1 \\ 2 & 0 \end{bmatrix}$, and let $B = P^{-1}AP$ (with A defined by h from part (a)). Compute B and prove that B and A have the same eigenvalues.