Your name:

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Math 22 Summer 2017, Homework 5, due Fri July 28 *Please show your work, and check your answers. No credit is given for solutions without work or justification.*

(1) Let $A = \begin{bmatrix} 5 & -1 & -3 & 11 \\ -2 & 1 & 0 & -5 \\ 3 & -2 & 1 & 8 \end{bmatrix}$.

(a) Find a basis for $\operatorname{Col} A$.

(b) What is the dimension of the subspace spanned by the columns of A ?

(c) Find a basis for $\operatorname{Nul} A$.

(d) Write, and explain, an inequality satisfied by the dimension of the nullspace of *any* matrix B formed by deleting rows from A. [Hint: what is a subspace of what?]

(2) Consider a linear transformation $T : V \to W$ where V is a vector space with basis $\mathcal{B} = \{\mathbf{b}_1, \ldots, \mathbf{b}_n\}$ and W is a vector space with basis $\mathcal{C} = \{\mathbf{c}_1, \ldots, \mathbf{c}_m\}$.

(a) Let $\mathbf{x}, \mathbf{y}$ be in V. Prove that $[\mathbf{x} + \mathbf{y}]_{\mathcal{B}} = [\mathbf{x}]_{\mathcal{B}} + [\mathbf{y}]_{\mathcal{B}}$. (Here you may use results up to and including the Unique Representation Theorem. Keep in mind V can be any vector space, not necessarily $\mathbb{R}^n$.)

(b) Suppose the map T is onto. What relation between n and m must hold? Prove your answer. [Hint: construct a map from $\mathbb{R}^n$ to $\mathbb{R}^m$ using coordinate maps, then prove that map is onto]

(3) (a) Recall the standard basis $\{1, t, t^2\}$ for $\mathbb{P}_2$. When we express these monomials about a new origin $t = a$, we get the set $\{1, (t-a), (t-a)^2\}$. Prove, for any a, that this set is also a basis for $\mathbb{P}_2$. [Hint: you many use that the coordinate map for the standard basis is an isomorphism.]

(b) The subspace $H = \text{Span}\left\{\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right\}$ is in the span of the vectors $\mathbf{b}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\mathbf{b}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. Explain whether $\{\mathbf{b}_1, \mathbf{b}_2\}$ is a basis for H.