Your name:

Instructor (please circle): Alex Barnett Michael Musty

Math 22 Summer 2017, Homework 4, due Fri July 21 *Please show your work, and check your answers. No credit is given for solutions without work or justification.*

(1) (a) Find the determinant of the matrix $A = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 2 & 0 & 4 & 7 \\ 1 & 0 & 5 & 4 \\ 17 & 2 & 38 & 99 \end{bmatrix}$

(b) Is A from above invertible, and why?

(c) Prove that, if A is any $n \times n$ matrix and P is an invertible $n \times n$ matrix, that $\det\,(PAP^{-1}) = \det\,A$. [Please state when and which properties of det you use.]

(2) Is each of the following sets a vector space? Explain what result(s) you used to prove your claim. You may assume that $\mathbb{R}^n$ is a vector space.

(a) $\left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix}, \quad a+b+c=0 \quad \text{and} \quad 3a-b+2c=0 \right\}$

(b) The set of vectors $\begin{bmatrix} 2 \\ 1 \end{bmatrix} + t \begin{bmatrix} 1 \\ 3 \end{bmatrix}$, where t takes all real values.

(c) The set of vectors $\begin{bmatrix} 2 \\ 1 \end{bmatrix} + t \begin{bmatrix} 6 \\ 3 \end{bmatrix}$, where t takes all real values.

(d) The set of functions of the form $a + bt^2$, where a and b are real. [Hint: relate this set to $\mathbb{P}_2$]

(3) Let $T : \mathbb{R}^4 \to \mathbb{R}^3$ be a linear transformation with standard matrix

$$A = \begin{bmatrix} 3 & 0 & 6 & -3 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & 2 & -1 \end{bmatrix}.$$

(a) Write the kernel of T (null space of A) in form of a span.

(b) Is Col A equal to $\mathbb{R}^3$? (explain)

(c) True/False: If the kernel of any linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ is a single point, then $n \geq m$? (explain, or, if false, correct the deduction)

BONUS Find a 2×2 matrix A for whom Col A and Nul A are equal (careful: equal as sets).