Your name:

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Math 22 Summer 2017, Homework 3, due Fri July 14 *Please show your work, and check your answers. No credit is given for solutions without work or justification.*

(1) Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation defined by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ -x_1 + 3x_2 \\ 3x_1 - 2x_2 \end{bmatrix}.$$

(a) Find the standard matrix for T.

(b) Is T one-to-one? Explain why or why not.

(c) Is T onto? Explain why or why not.

(2) Let $A = \begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & -4 \\ 1 & 0 & 2 \\ 0 & -2 & 1 \end{bmatrix}$.

(a) If A is not invertible, prove it, else, if A is invertible, find A^{-1}.

(b) Use the previous part of this problem to express the vector $\begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$ as a linear combination of the columns of A.

(c) Let A be an arbitrary 4×4 invertible matrix. Write a linear system involving the matrix A whose solution gives the third column of A^{-1}. (Write your linear system as a matrix equation).

(3) A certain disease affects the human population as follows over the course of one year: Two thirds of the initially healthy (H) humans become sick (S), while the rest remain healthy. Over that same year, one third of the initially sick recover and become healthy, one third remains sick, and one third sadly dies.

(a) Stacking the H and S populations into a column vector $\mathbf{x}$, write the population vector at year $k+1$ in terms of that at year k. Be sure to give the matrix. [Note: the dead are not included in the column vector nor the matrix.]

(b) If the populations at "year zero" are 900 healthy and zero sick, what are the populations at year 2? (ie, after two years of dynamics).

(c) Find a *matrix* that takes any population vector at year k to that at year $k+2$. [Hint: check against part b.]