Your name:

Instructor (please circle): Alex Barnett Michael Musty

Math 22 Summer 2017, Homework 1, due Fri June 30 *Please show your work, and check your answers. No credit is given for solutions without work or justification.*

(1) Given the system of equations, answer the following questions.

$$\begin{aligned} 2x_1 + 4x_3 &= 6 \\ -2x_1 + 3x_2 + 11x_3 &= 15 \\ -4x_1 + 3x_2 + 7x_3 &= 9 \end{aligned}$$

(a) Write the augmented matrix and transform it to reduced echelon form:

(b) Write the *general* solution to the linear system, if there is one:

(2) True or false (no working needed, just circle the answer):

(a) T / F: The matrix $\begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$ is in echelon form.

(b) T / F: If an echelon form of an augmented matrix has a row of all zeros (including the right-hand side), then the linear system must be consistent.

(c) T / F: Two linear systems that are row equivalent always reduce to the same reduced echelon form.

(d) T / F: Given any two vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^m$, it always holds that $\text{Span}\{\mathbf{v}, \mathbf{v}, \mathbf{u}\} = \text{Span}\{\mathbf{u}, \mathbf{v}\}$.

(e) T / F: A linear system with more equations than unknowns cannot have a unique solution.

(f) T / F: The span of two vectors in $\mathbb{R}^m$ is either a plane passing through the origin, a line passing through the origin, or the set consisting of the origin alone.

BONUS: Suppose two 1×2 linear systems have the same *solution sets*. Either prove that their augmented matrices have exactly the same reduced echelon form, or find a counterexample. [This is a challenge problem inspired by a student question; you may need more space!]

(3) For what value(s) of the real number h can the vector $\begin{bmatrix} 1 \\ -1 \\ -5 \end{bmatrix}$ be written as a linear combination of the three vectors $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \\ h \end{bmatrix}$, and $\begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$?

For what value(s) of h is the span of the set of three vectors equal to $\mathbb{R}^3$?