Math 22 X14 Extra credit homework

Proofwriting assignment #3

Directions: this homework is for extra credit, up to 2% extra credit over the whole course. This is a list of suggested exercises to work through; you are by no means required to complete them all. There is no due date for this homework; once you feel like you're finished you can drop it in the box outside Kemeny 008 (in the "Extra Credit" slot), email it to me or set an appointment with me to talk about it in person. If you need help with it please come directly to the instructor.

1. It's always good practice to go look at the proofs of all the theorems we've covered in class, understand them and try to replicate them without guidance. This is especially true for the properties of the determinant and what we said in class about similar matrices representing the same linear transformation in a different basis.

2. While we decided to take a computation approach to the determinant, it can also be seen as the unique multilinear alternating map on square matrices that takes the value 1 on the identity. Because our textbook was written with a computational approach in mind this view of the determinant is missing, however I can give you some sources to look at to understand the topic better.

3. This requires you to have at least picked up some notions from section 4.1, but it might be a good idea to go over sections 4.4 and 4.7 again, especially considering that we spent some time in class working through the $\mathbb{R}^n$ case. Section 5.4 also takes a whole new light when the notion of a vector space has been acquired. If you're interested in it let me know and we can figure out how to tailor the material to your interests.

4. If you're interested in the Jordan form, there is a lot of interesting theory behind it. You can look at the two pages `http://math.rice.edu/~friedl/math355_fall04/Jordan.pdf` and `http://math.berkeley.edu/~peyam/Math110Sp13/Handouts/Jordan%20Canonical%20Form.pdf`, but these give you only a view of the algorithm. There is a much deeper and richer theory of the Jordan form, that's usually talked about in a third course in abstract algebra (about what's called module theory). If you're interested we can work together in trying to give you a glimpse of what is actually going on with it, and why the Jordan form itself is more important theoretically than it is computationally.

5. The second paper on PageRank I posted (`http://www.math.dartmouth.edu/~m22x14/PageRank2.pdf`) is meant as a serious survey paper on the topic. As such, it contains many proofs and proof-based exercises that can teach you a lot about proofwriting. While the paper itself is on an applied math topic (and some of the stuff later in the paper is less interesting from a proofwriting perspective), it still contains some good examples of how proofs are written, as well as give you a deeper understanding of the theory behind Markov chains and diagonalization.

6. Try your hand at some proofs with the following exercises:

- Section 3.2: 31-36, 41-44
- Section 3.3: 25, 26, 29-32
- Section 5.1: 23-30
- Section 5.3: 23-32
- Section 5.4: 19-29
- Section 5.5: 22-26 (problems 25 and 26 prove the statement we made about the $\mathrm{Im}(v)$, $\mathrm{Re}(v)$ basis for a pair of complex conjugate eigenvalues)