Math 22 X14 Extra credit homework

Computational assignment #3

Directions: this homework is for extra credit, up to 2% extra credit over the whole course. This is a list of suggested exercises to work through; you are by no means required to complete them all. There is no due date for this homework; once you feel like you're finished you can drop it in the box outside Kemeny 008 (in the "Extra Credit" slot), email it to me or set an appointment with me to talk about it in person. If you need help with it please come directly to the instructor.

1. Implement an algorithm that outputs the determinant of an $n \times n$ matrix. There are various ways you can do this: you can try and brute-force an algorithm that computes the determinant along the first row, you can use the LU factorization to quickly get the determinant (shifting the computational burden on the LU algorithm), you can do row-reduction and keep track of the interchange and scalar multiple operations you're doing.

2. In a similar vein to the previous problem, once you have an algorithm for the determinant (or using the built-in algorithm) use it to compute the inverse of an $n \times n$ matrix using the cofactor matrix (see section 3.3)

3. Implement an algorithm that, given a matrix A, outputs the diagonalization matrices P, D or tells you A is not diagonalizable and why. You can synergize this part with the first problem of this assignment, using your algorithm to compute the characteristic polynomial of A and using some built-in function to find the roots.

4. You can look at chapter 5.6 and 5.7 for some concrete applications of diagonalization to real-world problems. Note that 5.7 requires you to be familiar with differential equations (in fact, it probably rehashes some notions you learned in your ODE class). You can also look at chapter 5.8 for some strategies to estimate eigenvalues iteratively, and implement those algorithms. Any problem at the end of those sections is good practice.

5. That's all for the stuff based on the book. A good challenge would be to understand the Jordan canonical form and how to get the "generalized eigenvectors" that make up the matrix P. More information can be found at `http://math.rice.edu/~friedl/math355_fall04/Jordan.pdf` (which has a nice algorithm for figuring out the blocks) and at `http://math.berkeley.edu/~peyam/Math110Sp13/Handouts/Jordan%20Canonical%20Form.pdf` (which I think does a better job in explaining how to find the generalized eigenvectors).

6. Regarding PageRank: you can look at this webpage (`http://www.math.cornell.edu/~mec/Winter2009/RalucaRemus/Lecture3/lecture3.html`) for some more examples of the algorithm, and it has a couple of toy problems at the bottom of the page to use the PageRank algorithm with. You can also look at the exercises in the PageRank

paper I posted (`http://www.math.dartmouth.edu/~m22x14/PageRank2.pdf`). Don't be scared by the excessive math lingo, just skip to the exercises and find the ones that ask you to explicitely compute stuff.

7. If you're looking for some problems that would test your algorithm, any problem in the book with the [**M**] symbol is meant to be solved with a [**M**]atrix program such as MATLAB or Mathematica. More specifically, good problems to work on at this stage are

    - Section 3.1: 43-46
    - Section 3.3: 33-35
    - Section 5.1: 37-40
    - Section 5.2: 23-30
    - Section 5.3: 33-36
    - Section 5.5: 27-28