Math 22: Linear Algebra. PRACTISE MIDTERM 1 ANSWERS

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No guarantee of correctness—please email with corrections.

1. Consistent, x_2 is free.

$$\begin{aligned} x_1 &= -5 - 3x_2 \\ x_2 &= x_2 \\ x_3 &= 1 \\ x_4 &= 2 \end{aligned}$$

2. consistent, 2 free vars

$$\begin{aligned} x_1 &= -5 + x_4 \\ x_2 &= -1 + x_3 + x_4 \\ x_3 &= x_3 \quad \text{free} \\ x_4 &= x_4 \quad \text{free} \end{aligned}$$

3. (a) see Ch. 1.7 p. 65
   (b) yes. 3 pivots
   (c) no since treating as aug matrix 2x1 with RHS, is inconsistent.
   (d) spans when pivot in every row, *i.e.* $h \neq 4$.

4. (a) False. You can be one-to-one but not be onto. *e.g.* $T : \mathbb{R}^2 \to \mathbb{R}^3$ given by $T(x, y) = (x, y, 0)$

   (b) linearity $T(c_1\mathbf{u}_1 + c_2\mathbf{u}_2) = c_1T(\mathbf{u}_1) + c_2T(\mathbf{u}_2)$ gives answer $\begin{bmatrix} 11 \\ 9 \end{bmatrix}$.

   (c) $ad - bc = 12 - (-12) = 0$ so not invertible.

   (d) True. Use properties of transpose and inverse.

   (e) the matrix formed by stacking the vectors is square, so if it misses a pivot in a row, it must also in a column, so they cannot by linearly independent.

5. $\mathbf{x} = \mathbf{p} + \alpha\mathbf{v}_1$ with $\mathbf{p} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$ and $\mathbf{v}_1 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$.

   The solution set is a line not through origin.

6. $\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$

   T is onto since every point in $\mathbb{R}^2$ can be reached by such a transformation from some point $\mathbf{x}$.