# Math 22: Linear Algebra. PRACTISE MIDTERM 1

No calculators. Please answer on this sheet. Your NAME:

1. Solve this system of linear equations using reduction of the augmented matrix to reduced echelon form:

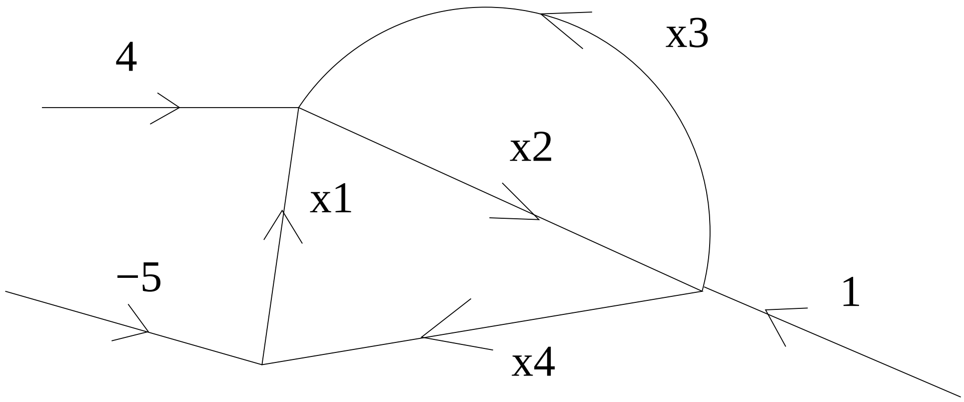
$$\begin{array}{rrrrcr} & & 3x_3 + & x_4 & = & 1 \\ 2x_1 + & 6x_2 + & x_3 - & 2x_4 & = & 15 \\ -x_1 - & 3x_2 + & 2x_3 + & x_4 & = & -5 \end{array}$$

If consistent, write the general solution:

$$\begin{aligned} x_1 &= \\ x_2 &= \\ x_3 &= \\ x_4 &= \end{aligned}$$

Otherwise, if not consistent, explain why.

2. Write the general solution to the following traffic flow problem, if there is one.



3. (a) Define the concept of linear independence.

(b) Fixing $h = 0$, Is the set of three vectors $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ h \end{bmatrix}$, linearly independent?

(c) Still fixing $h = 0$, is the last of these vectors in the span of the first two vectors?

(d) What *condition* on $h$ is required if the set of three vectors is to span $\mathbb{R}^3$?

4. (a) True/false? If the range of a linear transformation $T$ does not fill the codomain, then $T$ must not be one-to-one. If true, explain. If false, find a counterexample.

(b) If a linear transformation takes $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ to $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ to $\begin{bmatrix} -5 \\ -1 \end{bmatrix}$, is it possible to know to what vector does it take $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$? If so, find the answer. In either case Explain why using any relevant definition or theorem.

(c) Is the matrix $\begin{bmatrix} 2 & -3 \\ -4 & 6 \end{bmatrix}$ invertible? If so, give the inverse.

(d) True/false? $A^T(A^{-1})^T = I$, the identity, for *all* invertible matrices $A$?

(e) If a set of $m$ vectors in $\mathbb{R}^m$ do not span $\mathbb{R}^m$, what (if anything) can you always say about their linear independence?

5. For $A = \begin{bmatrix} 5 & 10 \\ 2 & 4 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 15 \\ 6 \end{bmatrix}$, write the solution set of $A\mathbf{x} = \mathbf{b}$ in parametric vector form $\mathbf{x} = \mathbf{p} + \alpha\mathbf{v}_1 + \beta\mathbf{v}_2 + \cdots$

What type of geometrical object (within the domain $\mathbb{R}^2$) is the solution set?

6. A transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ corresponds to a rotation of points by angle $-\pi/2$ (*i.e.* , $90^\circ$ *clockwise*), followed by a magnification (dilation, or rescaling) by factor 2. Find the *standard matrix* for $T$:

Is $T$ 'onto' $\mathbb{R}^2$ ? (You may explain graphically or algebraically).

Reminder: material is everything up to an including 2.3 (note: I have decided not to include 2.5 since we didn't finish it all on Fri).

More practise problems (sorry if some duplicate HW problems):
**1.1:** 15, 28.
**1.2:** 15, 19.
**1.3:** 13.
**1.4:** 11, 14, 18, 19, 24.
**1.5:** 5, 9, 10, 29–32.
**1.6:** 13.
**1.7:** 2, 5, 15, 17, 27, 28.
**1.8:** 3, 9, 11.
**1.9:** 2, 9, 10, 17, 25.
**1.10:** Any of the remaining 2x2 migration matrix probs without an [M] symbol.

Chapter 1 supplementary exercises (p. 102): 1, 4.

**2.1:** 8, 10.
**2.2:** 3, 6, 32.
**2.3:** 5, 15, 17, 19, 24.