Math 22: Linear Algebra. PRACTISE FINAL ANSWERS

August 24, 2006

1. (a) W is Col A for $A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & -1 & 1 \\ 1 & 0 & 3 \end{bmatrix}$. A column space is the span of some vectors, so must be a subspace.

   (b) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix} \right\}$

   (c) $W^{\perp}$ is the set of all vectors in $\mathbb{R}^3$ that are orthogonal to all vectors in W.

   (d) Either find basis for Nul A^T, by writing A^T and following usual method. Or find the single vector orthog to the above two by subtracting orthogonal projection onto W. Answer: $\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \right\}$

2. (a) the set is reached by addition of a const vector (any soln of inhomog eqn).

   (b) check : lin indep ? (yes). Does B span W? (ie is each vector in W in span B ? yes since the two vectors comprising W are in span B). So, yes.

   (c) no since not lin indep. (even though do span W).

3. (a) $8 - 3 = 5$.

(b) No since always have 2 or more free vars. So solution set is a constant plus a null-space, which is generally not a subspace since zero vector not included.

(c) See book p. 65.

4. (a) $M = \begin{bmatrix} 0.5 & 1 \\ 0.5 & 0 \end{bmatrix}$.

(b) $(0.75, 0.25)$

(c) $(2/3, 1/3)$.

(d) M is regular, since although it has a zero entry, M has all entries ¿ 0, important to check this! Only then can the theorem (p. 294) stating convergence to the steady state be used.

5. (a) eigvals are 2 twice, both magnitude > 1, so repellor.

(b) -4, 6, both magnitude > 1, so repellor.

(c) $1/2, 3/2$, saddle point.

(d) 6 largest in magnitude, corresp eigvec is $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

6. (a) yes since 2 distinct eigvals. $A = PDP^{-1}$, $D = \begin{bmatrix} 1/2 & 0 \\ 0 & 1 \end{bmatrix}$, $V = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$.

(b) $PD^kP^{-1} = \begin{bmatrix} 1 & 1-2^{-k} \\ 0 & 2^{-k} \end{bmatrix}$.

7. (a) Symmetric matrices are always diagonalizable, and orthogonally so. $D =$ diagonal with eigenvalues 2,3,12. $P = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{3} & -1/\sqrt{6} \\ -1/\sqrt{2} & 1/\sqrt{3} & -1\sqrt{6} \\ 0 & 1/\sqrt{3} & 2/\sqrt{6} \end{bmatrix}$. I think!

(b) Since cols of P are orthonormal, coeffs $\alpha_i = \mathbf{x} \cdot \mathbf{v}_i$ for $i = 1, 2, 3$. So $[\mathbf{x}]_B = \begin{bmatrix} -1/\sqrt{2} \\ \sqrt{3} \\ -3/\sqrt{6} \end{bmatrix}$.

8. (a) See problem 9 of
   http://www.math.dartmouth.edu/~m22f05/math_22_final_practice_sol.pdf
   (b) See problem 10 of the same, for nice solutions.

9. see p. 62.