Math 22 Lin Alg: Homework 7

due Wed Aug 9 ... but best if do relevant questions after each lecture

As per some of your evaluation requests, everything on this HW is relevant for Midterm 2, so you can work on understanding it without any conflicted feelings! Lots of eigen-stuff this week, very important for applications.

4.6: (see goals from last time)

3 (carried over from HW6), 14 (carried over from HW6), 16 (please read carefully, and explain), 18 (more careful reading!), 20, 24.

5.1: Goals: Know what eigenvalues and eigenvectors are and how to find them; be able to compute the basis for the eigenspace of a matrix corresponding to a given eigenvalue.

2, 3, 7, 14, 18, [Hint: recognize the special form], 22 (remember only say 'True' if it's *always* true), 24 (this is a fun challenge. Don't give a multiple of the identity. Check your answer doesn't have a second eigenvalue. What is the general form?)

5.2: Goals: Understand the relationship between the eigenvalues of a matrix and the characteristic polynomial; be able to compute characteristic polynomials; know the definition of similar matrices.

7, 10 (remember to expand and simplify by gathering into powers of λ, like in the examples), 16, 19 (A proof one: apply the definition given of the characteristic poly; when you get it you'll kick yourself!).

A. How long does it take your computer to find the eigenvalues of a 100×100 matrix? A 300×300 matrix? A $10^3 \times 10^3$ matrix? Use random (nonsymmetric) matrices each time, and use the `eig` command. To get accurate timings you may want to use the `tic` and `toc` commands on a single line, thus:

```
tic; D = eig(A); toc
```

1. Use this to make a conjecture on the amount of work required for a $n \times n$ matrix—what power of n does it follow (roughly)? How does this compare to row reduction (*i.e.* solving $A\mathbf{x} = \mathbf{b}$)?
2. Roughly how many times faster or slower is finding the eigenvalues if the random matrix is now *symmetric*? The definition of symmetric kicks off section 7.1 in book (I leave it up to you how to make a random symmetric matrix... how?). Matlab uses a different algorithm for symmetric vs nonsymmetric matrices.
3. *Estimate* (do not try!) the amount of time your computer would take to find the eigenvalues of a $10^6 \times 10^6$ nonsymmetric matrix. Please express your answer in sensible time units that a 'person on the street' would find informative.

In fact other algorithms exist for such large matrices, which are routine in engineering vibration problems, that are *much* faster when the matrix has a large fraction of zero entries!