Math 22 HW 6

you should observe this numerically by seeing det $(A^TA) \approx 10^{-16}$ (ie v. close to zero, within rounding error).

3.2 45 (3 points) A^TA has $m<n$ pivots; AA^T has m pivots $(m<n)$

So A^TA is not invertible but AA^T is. — but $\det(AA^T) \sim 10^{-2}$, not that small..

4.2 39 (3 points) (a) $A \sim \begin{bmatrix} 1 & 0 & 1/3 & 0 & 10/3 \\ 0 & 1 & 1/3 & 0 & -26/3 \\ 0 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$. col # 1,2,4 are pivot columns of A, thus $\{a_1, a_2, a_4\}$ spans the column space.

(b)
$$Nul(A) = \begin{pmatrix} -1/3 \\ -1/3 \\ 1 \\ 0 \\ 0 \end{pmatrix} x_3 + \begin{pmatrix} -10/3 \\ 26/3 \\ 0 \\ 4 \\ 1 \end{pmatrix} x_5$$

(c) One-to-one $\Rightarrow Nul(A) = 0$; onto $\Rightarrow Col(A) \neq \mathbb{R}^4$.

4.3 3 (2 points) The vectors are linearly dependent and does not span $\mathbb{R}^3$.

13 (2 points) Basis for Nul A: $\begin{bmatrix} -6 \\ -5/2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ 3/2 \\ 0 \\ 1 \end{bmatrix}$; Basis for Col A: $\begin{bmatrix} -2 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ -6 \\ 8 \end{bmatrix}$.

22 (3 points) a. False. see def. of a basis.

b. True, by the spanning set Theorem

c. True. see the subsection "Two Views of a Basis".

d. False. See two paragraphs before example 8.

e. False. See the warning after theorem 6.

24 (2 points) Let $A = [v_1 \cdots v_n]$. Since A is square and its columns are linearly independent, its columns also span $\mathbb{R}^n$ by the Invertible Matrix Theorem

So $\{v_1, \cdots, v_n\}$ is a basis for $\mathbb{R}^n$.

4.4 12 (3points) $\begin{bmatrix} -7 \\ 5 \end{bmatrix}$

30 (3points) Linearly dependent since the coordinate vectors are linearly dependent.

4.5 8 (2points) $\begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$, dim is 3.

22 (3points) No Laguerre polynomial is a linear combination of the Laguerre polynomials of lower degree.
By Theorem 4 (4.3), the set of polynomials is linearly independent.
Since this set contains four vectors and $\mathbb{P}_3$ is four-dimentional, the set is a basis of $\mathbb{P}_3$ by the Basis Theorem.

4.6 8 (2points) dim Nul $A = 2$.
It is impossible for Col A to be $\mathbb{R}^4$ since the vectors in Col A have 5 entries.
Col A is a four-dimensional subspace of $\mathbb{R}^5$.