HW 5

Math 22

answers claiming rounding error is the cause are ~~wrong~~ : you can check $Ax=b$ to high accuracy

A (5 points)

1. $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$ 2 pivots

note $Ax=b$ consistent even if A^{-1} doesn't exist!

2. $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

3. $\det(A)=0$.

inv(A): should produce an answer in which each entry is around the size of 10^{16}.

4. Matlab might still produce an answer but ~~those are only due to rounding errors,~~ ~~therefore it may not be correct.~~ since $Ax=b$ is not unique, you may not get $[1;0;0]$.

B (4 points)

(with differences decaying exponentially)

1° Tends towards $\begin{bmatrix} 4/3 \\ 2/3 \end{bmatrix}$

2° Tends towards $\begin{bmatrix} 4/3 \\ 2/3 \end{bmatrix}$.

interesting that this does not depend on starting vector. We'll learn why soon!

3.1 10 (3 points) -6. (Start with 2^{nd} row).

3.2 8 (3 points) 0

34 (2 points) $\det(PAP^{-1}) \overset{(Th.6)}{=} (\det P)(\det A)(\det P^{-1}) = (\det P)(\det A)(\det P)^{-1} = \det A$.

4.1 2 (2 points) a. Given $\begin{bmatrix} x \\ y \end{bmatrix}$ in W and any scalar c, the vector $c\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} cx \\ cy \end{bmatrix}$ is in W because $(cx)\cdot(cy) = c^2(xy) \geq 0$, since $xy \geq 0$.

b. Example: Let $u=\begin{bmatrix} -1 \\ -7 \end{bmatrix}$, $v=\begin{bmatrix} 2 \\ 3 \end{bmatrix}$, then $u, v \in W$, $u+v \notin W$.

8 (3 points) Yes. The zero vector is in the set.

If $p, q \in H$ then $(p+q)(0) = p(0)+q(0) = 0$, $\therefore p+q \in H$.

For any scalar c, $(cp)(0) = c\,p(0) = c\cdot 0 = 0$, $\therefore cp \in H$.

4.1 16 (2points) Not a vector space since the zero vector is not in W.

24 (3points) a. True. See definition of a vector space.

b. True. see statement (3) before Example 1.

c. True. See the paragraph before example 6.

d. False. See example 8.

e. False. (ii) and (iii) are stated incorrectly.

e.g., in (ii), there is no statement of that u, v represents all possible elements of H.

4.2 6 (2points) $\begin{bmatrix} -6 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 8 \\ -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}.$

23 (3points) w is in both Nul A and Col A.