Math 22 Lin Alg: Homework 5

due Wed Jul 26   . . . but best if do relevant questions after each lecture

Some of your homework time this week is devoted to getting started with Matlab (a versatile and powerful package which I promise will bring you joy and success in your future careers), or an alternative if you are already used to one (Maple, R, Mathematica, octave, etc). Either install on your own computer or find a computer lab with it there. For Matlab help, always start with our course website http://math.dartmouth.edu/∼m22x06, then ask friends, myself, or Susan A. Schwarz.

Please read 'Simple intro from Utah' linked on Resources of the course page, and browse the other links briefly.

Let's see how Matlab can do some of our familiar operations. (Question A is worth more than a usual HW question since it has many parts).

A. Enter the matrix `A = [1 2 3; 4 5 6; 7 8 9]`

   1. Try `rref(A)` to reduce to REF (how many pivots does it have?).
   2. Try `null(A, 'r')` to produce the list of vectors that appear in the parametric vector form for the solution to $A\mathbf{x} = \mathbf{0}$.
   3. Try `det(A)` to compute determinant. Why do you expect this? Try `inv(A)`. Scary, huh? What do you think is happening?
   4. Enter the column vector `b = [1;4;7]`. Solve $A\mathbf{x} = \mathbf{b}$ by typing `x = A\b`. Why does this differ from the expected 'easy' answer (which you can get by inspection)? Then compute `A*x`. Does it equal `b`? What's going on?

B. Imagine a population model has the migration matrix $A = \begin{bmatrix} 0.9 & 0.2 \\ 0.1 & 0.8 \end{bmatrix}$. Enter this into Matlab, and the starting vector $\mathbf{x}_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Evolve your populations to get $\mathbf{x}_1$, $\mathbf{x}_2$, etc, up to at least 20 years, either by entering `x = A*x` repeatedly (use up arrow to repeat last command), or using `for` loop described in the Utah intro. What is happening? Try the same for the starting vector $\mathbf{x}_0 = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$. How does this differ from before?

   From now, you might want to use Matlab to check your answers. Please continue to show your working as you do it by hand.

3.1: (see Goals listed in HW4)

   10, 12 (this shows you a lovely property of triangular matrices).

3.2: Goals: understand the connection between determinant, diagonal pivots, and invertibility. Be able to compute determinant using row reduction.

   8, 34 (careful: you are not allowed to just swap A and P in the string of matrices PAP^{-1}, since they won't commute in general), 39.

4.1: Goals: Know definitions of a vector space and subspace, be able to check if a set of objects form a vector space or subspace.

   2, 8, 12, 16, 24.

4.2: Goals: Be able to computer the null space and column space of a matrix, understand how they are related and why they are important.

   6, 15, 23.