HW 1 — Math 22 Lin. Alg

Answer key.

1.1 12 (3 points)

$$\begin{bmatrix} 1 & -3 & 4 & -4 \\ 3 & -7 & 7 & -8 \\ -4 & 6 & -1 & 7 \end{bmatrix} \xrightarrow[3+4\times 1]{2+(-3)\times 1} \begin{bmatrix} 1 & -3 & 4 & -4 \\ 0 & 2 & -5 & 4 \\ 0 & -6 & 15 & -9 \end{bmatrix} \xrightarrow{3+3\times 2} \begin{bmatrix} 1 & -3 & 4 & -4 \\ 0 & 2 & -5 & 4 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

Inconsistent

20 (2 points)

$$\begin{bmatrix} 1 & h & -3 \\ -2 & 4 & 6 \end{bmatrix} \xrightarrow{2+2\times 1} \begin{bmatrix} 1 & h & -3 \\ 0 & (2h+4) & 0 \end{bmatrix}$$

If $h=-2$, then x_2 independent variable. The system is consistent.

If $h\neq-2$, then $x_2=0$. The system is consistent.

Thus the system is consistent for all $h \in \mathbb{R}$.

24 (3 points)

a. True. see p. 8

b. False.

c. False

d. True see p. 3

1.2

2 (3 points)

a. Reduced echelon form. consistent but not unique.

b. Echelon. consistent and unique.

c. Not echelon and not consistent.

d. Echelon and not consistent.

4 (3 points)

$$\begin{bmatrix} 1 & 3 & 5 & 7 \\ 3 & 5 & 7 & 9 \\ 5 & 7 & 9 & 1 \end{bmatrix} \xrightarrow[3+(-5)\times 1]{2+(-3)\times 1} \begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & -4 & -8 & -12 \\ 0 & -8 & -16 & -34 \end{bmatrix} \xrightarrow[3\times\frac{-1}{2}]{2\times\frac{-1}{4}} \begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & 1 & 2 & 3 \\ 0 & 4 & 8 & 17 \end{bmatrix}$$

$$\xrightarrow{3+(-4)\times 2} \begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 5 \end{bmatrix} \xrightarrow{3\times\frac{1}{5}} \begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{2+3\times(-3)} \begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{1+(-3)\times 2+(-7)\times 3} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Pivot cols 1, 2, and 4.

← note you can row reduce the last column too!

1.2

12 (3 points)

$$\begin{bmatrix} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ -1 & 7 & -4 & 2 & 7 \end{bmatrix} \xrightarrow{③+①+4\times②} \begin{bmatrix} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{cases} x_1 = 7x_2 - 6x_4 + 5 \\ x_2 \text{ is free} \\ x_3 = 2x_4 - 3 \\ x_4 \text{ is free} \end{cases}$$

1.3

8 (2 points)

$w = 2v - u$, $x = -2u + 2v$, $y = -2u + 3.5v$, $z = -3u + 4v$

Every vector in $\mathbb{R}^2$ is a linear combination of u and v.

don't forget to answer!

12 (2 points)

$$\begin{bmatrix} 1 & 0 & 2 & -5 \\ -2 & 5 & 0 & 11 \\ 2 & 5 & 8 & -7 \end{bmatrix}$$ is inconsistent, thus b is not a linear combination of a_1, a_2, a_3.

24 (3 points)

a. True. see p.31

b. True see p.32

c. False. see p.32

d. True. see p.35

e. True. see p.35

32 (2 points)

The solution is not unique. The solutions that arise from letting $x_1=0$ and $x_2=0$ respectively are different.

1.2
33 (2 points + 1 bonus point)

$p(t) = 7 + 6t - t^2$

generalization: you need $n+1$ values of t to determine a nth order polynomial.