Ungraded Quiz + Questionnaire 3

Your name: __________

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1. Let $V = \mathbb{P}_2$ and let $B = \{1, t, t^2\}$ be the standard basis for $\mathbb{P}_2$. Compute $[1 - 4t + 7t^2]_B$.
   If $1 - 4t + 7t^2 = c_1(1) + c_2(t) + c_3(t^2)$ then
   $$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \\ 7 \end{bmatrix}.$$
   This is the B-coordinate vector $[1 - 4t + 7t^2]_B$.

2. **True** or false: if B and C are (finite) bases for the vector space V, then the change of basis matrix $C \overset{P}{\leftarrow} B$ is invertible. (The inverse is the change of basis matrix going in the other direction, $B \overset{P}{\leftarrow} C$.)

3. Compute the rank of the matrix
   $$A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \\ -4 & -8 \end{bmatrix}.$$
   We can row reduce by subtracting multiples of row 1 from the other two rows.
   $$A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \\ -4 & -8 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$
   There is one pivot column so the rank of the matrix is 1.

4. Is $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ an eigenvector of the matrix $A = \begin{bmatrix} 2 & -2 \\ -7 & 7 \end{bmatrix}$? If so, find the corresponding eigenvalue.
   It is an eigenvector with eigenvalue 0.
   $$A\mathbf{v} = \begin{bmatrix} 2 & -2 \\ -7 & 7 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = 0\mathbf{v}.$$
   Note that it is entirely okay for the eigen*value* to be 0, so long as the eigen*vector* $\mathbf{v}$ is nonzero.