Math 22: Linear Algebra with applications
Fall 2019 - Quiz 1

Date: 10/30/19

True false questions *(2 points each)*

1. If a multiple of one row of A is added to another row to produce a matrix B then $\det(A) = \det(B)$.
   ○ **True**   ○ **False**

2. If two columns of A are interchanged to produce a matrix B then $\det(A) = \det(B)$.
   ○ **True**   ○ **False**

3. If A is a 3×3 matrix then $\det(k \cdot A) = k^3 \det(A)$.
   ○ **True**   ○ **False**

4. If V is a vector space and H and K are subspaces, then $H + 3 \cdot K$ is a subspace.
   ○ **True**   ○ **False**

5. If A is an $m \times n$ matrix then the null space $Nul(A)$ of A is $\mathbb{R}^n$.
   ○ **True**   ○ **False**

6. If A is an invertible 5×5 matrix then $\dim Col(A) = 5$.
   ○ **True**   ○ **False**

7. If $p_1(t) = 1$, $p_2(t) = 2t$ and $p_3(t) = 3 - t$, then $\{p_1, p_2, p_3\}$ are linearly independent in $\mathbb{P}_3$.
   ○ **True**   ○ **False**

8. If a set $\{v_1, v_2, ..., v_p\}$ of vectors spans the vector space V and if S is a set of more than p vectors in V, then S is a linearly dependent set.
   ○ **True**   ○ **False**

9. If $T : \mathbb{R}^5 \to \mathbb{R}^3$ is a linear transformation, then $\dim(T(\mathbb{R}^5)) \leq 2$.
   ○ **True**   ○ **False**

Long answer question

1.) *(6 points)* Let

$$P = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix} \text{ and } \mathbf{v}_1 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}, \ \mathbf{v}_2 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}.$$

a) Find a basis $U = \{\mathbf{u}_1, \mathbf{u}_2\}$ for $\mathbb{R}^2$, such that $P = \underset{V \leftarrow U}{P}$ is the change-of-coordinates matrix from U to V.

b) Find a basis $W = \{\mathbf{w}_1, \mathbf{w}_2\}$, such that $P = \underset{W \leftarrow V}{P}$ is the change-of-coordinates matrix from V to W.