Math 22 - Fall 2019
Practice Exam 1

Your name: ______________________________

Section (please check the box): ☐ Section 1 (10 hour) ☐ Section 2 (2 hour)

INSTRUCTIONS

- Except on clearly indicated short answer problems, you must explain what you are doing, and show your work. You will be *graded on your work*, not just on your answer.
- It is fine to leave your answer in a form such as $\sqrt{239}$ or $(385)(13^3)$. However, if an expression can be easily simplified (such as $\cos(\pi)$ or $(3-2)$), you should simplify it.
- You may use the last page for scrap paper.
- This is a closed book exam. You may not use notes, computing devices (calculators, computers, cell phones, etc.) or any other external resource.

GOOD LUCK!

(1) Please indicate whether the following statements are **TRUE** or **FALSE**.
Circle the correct answer. You do not have to show your work, however thinking about the problem on a scrap paper is recommended.

1.) The linear system

$$\begin{aligned} 3x + 7y + z &= -1 \\ -x - y + z &= -3 \\ -2y - 2z &= 5 \end{aligned}$$

is consistent.

TRUE **FALSE**

2.) A vector $\mathbf{b}$ is in the span of the columns of a matrix A if and only if the equation $A\mathbf{x} = \mathbf{b}$ is consistent.

TRUE **FALSE**

3.) If there is a pivot in each row of a coefficient matrix, there are no free variables.

TRUE **FALSE**

4.) If there is a pivot in each row of an augmented matrix, the system is consistent.

TRUE **FALSE**

5.) A set of 3 vectors in $\mathbb{R}^4$ is always linearly independent.

TRUE **FALSE**

6.) A set of 6 vectors in $\mathbb{R}^7$ can never span $\mathbb{R}^7$.

TRUE **FALSE**

(2) Let

$$A = \begin{bmatrix} 3 & 0 & 3 & 0 \\ 1 & 2 & 3 & -4 \\ 2 & -1 & 1 & 2 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 6 \\ 0 \\ 5 \end{bmatrix}.$$

a) Solve the matrix equation $A\mathbf{x} = \mathbf{b}$, and write the solution in parametric vector form.

b) Without doing any row operations, write the solution set to the matrix equation $A\mathbf{x} = 5\mathbf{b}$.

c) Write the solution to the homogenous equation $A\mathbf{x} = \mathbf{0}$ in parametric vector form.

d) Is the solution set of $A\mathbf{x} = \mathbf{0}$ a point, a line, a plane, a 3-dimensional space, or all of $\mathbb{R}^4$? Explain your answer.

(3) We want to find a polynomial $p(x)$ in the xy-plane that passes through the following points in the plane:

$$P_1 = (-2, 2)\,,\ P_2 = (-1, 3)\,,\ P_3 = (0, -2)\,,\ P_4 = (1, 0)\ \text{ and }\ P_5 = (2, 6).$$

We know that $p(x)$ is a polynomial of the following form.

$$p(x) = a + bx^2 + cx^3 + dx^5 + ex^6.$$

a) Write down the equations for the coefficients of p.

b) Write down the augmented matrix of the system of linear equations from **a)**.
You do **not** have to solve this system.

(4) Let

$$\mathbf{v}_1 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \quad \mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \quad \mathbf{v}_3 = \begin{bmatrix} 3 \\ 1 \\ h \end{bmatrix} \quad \text{and} \quad \mathbf{v}_4 = \begin{bmatrix} -2 \\ 2 \\ 0 \end{bmatrix}$$

a) Find all the real values h for which $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ spans all of $\mathbb{R}^3$. Explain your answer.

b) Let $h = 1$, and write $\mathbf{v}_4$ as a linear combination of $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

(5) Determine if each set is linearly dependent or linearly independent. Justify your answer.

a) $\left\{ \begin{bmatrix} 1 \\ 5 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \end{bmatrix} \right\}$

b) $\left\{ \begin{bmatrix} -3 \\ 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ 3 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 3 \\ 2 \end{bmatrix} \right\}$

c) $\left\{ \begin{bmatrix} 10 \\ -4 \\ -1 \\ -6 \end{bmatrix}, \begin{bmatrix} 6 \\ 0 \\ 9 \\ -3 \end{bmatrix}, \begin{bmatrix} -7 \\ -10 \\ -2 \\ -9 \end{bmatrix}, \begin{bmatrix} 7 \\ -8 \\ 4 \\ 8 \end{bmatrix}, \begin{bmatrix} 3 \\ -5 \\ 1 \\ 5 \end{bmatrix} \right\}.$

(6) Let T be the linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} \frac{3}{2}x_1 + \frac{1}{2}x_2 \\ x_1 - x_2 \\ x_2 \end{bmatrix}.$$

a) Find the standard matrix for T.

b) Find a vector $\mathbf{x}$ whose image under T is $\begin{bmatrix} 5 \\ 2 \\ 1 \end{bmatrix}$.

c) Is T one-to-one? Justify your answer.

(7) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that first performs a horizontal shear that transforms $\mathbf{e}_2$ into $\mathbf{e}_2 - 2\mathbf{e}_1$ (leaving $\mathbf{e}_1$ unchanged) and then reflects points through the line $x_2 = -x_1$.

Find the standard matrix of T.

(8) Suppose that T is a one-to-one linear transformation and that $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p$ are linearly independent vectors in $\mathbb{R}^n$. Prove that $\{T(\mathbf{v}_1), T(\mathbf{v}_2), \ldots, T(\mathbf{v}_p)\}$ is also linearly independent.

(This page is intentionally left blank in case you need extra space for any of the problems. **If you use this page for a particular problem, it is essential that you make a note on the page where the problem appears, indicating that your work is continued here.)**