Matrix Labyrinth - a game inspired by Linear Algebra

Description of the game

Note: For more information about the math behind this game, read
- "*The math behind these games*" on the project webpage of our course.
- Lay, D. et al: *Linear Algebra and its applications*, **Chapter 2.1** and **3.1**.

In practice all calculations can be done with numbers, where $\emptyset = 0$, $\bullet = 1$ and $\circ = 2$. The corresponding operations are $+$: addition modulo 3 and $\cdot$: multiplication modulo 3.
A high score list is optional. The final design does not have to look like pieces on a Go board.

Aim: This game is inspired by the board game "Crazy Labyrinth". A 12×12 board consists of thirty six 2×2 squares with "random" values. On random 6 of white fields of the squares are items with labels 1 to 6. The aim is to collect all 6 items in the given order. A player starts on a white field in the left bottom corner. By manipulating the board by applying matrices to the board, he can create paths. The aim of the game is to move along these paths and collect the items.

Rules: During a move the player does the following two actions in the given order, where an action can also be skipped:

1.) **Multiply** four of the following matrices to any square of the board by multiplication either from the left or the right:
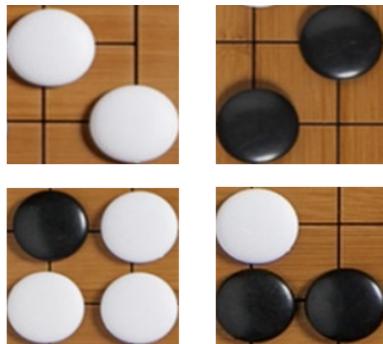
FIGURE 1. The first matrix (top left) inverts colors, the second (top right) swaps rows or columns. The result of the remaining two matrices is more complicated.

Here in each game the last two matrices are random matrices with non-zero determinant (see **Inititalization**), that should be the same for all players.

2.) **Move** vertically and / or horizontally along a path of white fields of the board.

Implementation: The game should be implemented with levels, a timer, a score and a high score list. The difficulty should increase over time by reducing the time in a new level. The game ends when the time is up. A two (to four) player game with more items is also possible. Then by obtaining the items the players gain the points of the items. The player with the highest score wins.
A practice level without time or where the matrix product can be practiced could be added. Interesting levels or maps could be saved.

Layout: The board should be in the middle and the timer and score on top. On the left hand side next to the board there should be the player's matrices and controls. The controls should be Push , Multiply Left , Multiply Right (Multiplication from the left and from the right), Move (with controls) and Undo (deselects a matrix or choice). Another button End turn indicates the end of the movement of the player.

Initialization: To ensure that no matrix vanishes (becomes a zero matrix) and that paths can be created during the game, the matrices have to be initialized in the following way: a 2×2 matrix is initialized with two random white pieces. Then the remaining two fields are filled with any of the three possible pieces ($\emptyset = 0, \bullet = 1, \circ = 2$), such that the matrix has non-zero determinant det (see book, **Chapter 3.1**), where for a matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \quad \text{we have} \quad \boxed{\det(A) = a \cdot d - b \cdot c}.$$

The last condition can be verified by choosing the last, fourth entry of the matrix accordingly.