Your name:

Instructor (please circle): Samantha Allen Angelica Babei

Math 22 Fall 2018 Homework 6, due Fri Oct 26 4:00 pm in homework boxes in front of Kemeny 108 *Please show your work, and check your answers. No credit is given for solutions without work or justification.*

(1) Consider the matrix $A = \begin{bmatrix} 2 & -4 & 8 & 2 \\ -1 & 3 & -3 & 0 \\ 1 & -1 & 5 & 2 \end{bmatrix}$.

(a) Find a basis for Row$A$.

$$\begin{bmatrix} 2 & -4 & 8 & 2 \\ -1 & 3 & -3 & 0 \\ 1 & -1 & 5 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & 4 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & 4 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Basis $B = \left\{ \begin{bmatrix} 1 \\ -2 \\ 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\}$

(b) Find the rank of A and the dimension of Nul$A$.

rank $A = 2$

dim Nul $A = 3-$ rank $A = 3 - 2 = 1$.

(2) True or false (no working needed, just circle the answer):

(a) T : A coordinate mapping is both one-to-one and onto.

(b) T : If $\dim V = 10$, then there exists a spanning set of 11 vectors in V.

(c) F: If the null space of a 5×6 matrix A is 4-dimensional, the dimension of the column space of A is 1.

(d) F: If the rank of a matrix A is equal to the number of columns of A, then A is an invertible matrix.

(e) F: If V is an n-dimensional vector space and S is a subset of V consisting of n vectors, then S is a basis for V.

(3) The set $B = \{1 - t^2, t - t^2, 2 - 2t + t^2\}$ is a basis for $\mathbb{P}_2$, the vector space of polynomials of degree at most 2.

(a) Find the change-of-coordinates matrix from B to the standard basis $C = \{1, t, t^2\}$ for $\mathbb{P}_2$.

$$\underset{C \leftarrow B}{P} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -2 \\ -1 & -1 & 1 \end{bmatrix}$$

Check:

$$\underset{C \leftarrow B}{P} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -2 \\ -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} a + 2c \\ b - 2c \\ -a - b + c \end{bmatrix}$$

$$a(1 - t^2) + b(t - t^2) + c(2 - 2t + t^2) = (a + 2c)1 + (b - 2c)t + (-a - b + c)t^2$$

(b) Find the coordinate vector of $\mathbf{p}(t) = 3 + t - 6t^2$ relative to B. Need to solve:

$$a + 2c = 3$$
$$b - 2c = 1$$
$$-a - b + c = -6$$

(OR use inverse of above matrix)

$$[\mathbf{p}]_B = \begin{bmatrix} 7 \\ -3 \\ -2 \end{bmatrix}$$