Your name:

Instructor (please circle): Samantha Allen Angelica Babei

Math 22 Fall 2018 Homework 5, due Fri Oct 19 4:00 pm in homework boxes in front of Kemeny 108 *Please show your work, and check your answers. No credit is given for solutions without work or justification.*

(1) In this exercise, let $W \subset \mathbb{R}^3$ be the set of all vectors of the form shown, where a, b, c represent arbitrary real numbers. In each case, determine if W is a subspace of $\mathbb{R}^3$. If yes, find a set S of vectors that spans W. If not, find a property of subspaces that W does not satisfy, and show why W does not satisfy it.

(a) $\begin{bmatrix} a-b \\ b+2 \\ -2a \end{bmatrix}$

Solution. W is not a subspace, since it doesn't contain $\mathbf{0}$: the system

$$\begin{bmatrix} a-b \\ b+2 \\ -2a \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

is not consistent. If the first and third rows are zero, then $a = b = 0$, in which case $\begin{bmatrix} a-b \\ b+2 \\ -2a \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$

(b) $\begin{bmatrix} a-b \\ 3b-2c \\ 2a+3c \end{bmatrix}$

Solution. $\begin{bmatrix} a-b \\ 3b-2c \\ 2a+3c \end{bmatrix} = a\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + b\begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix} + c\begin{bmatrix} 0 \\ -2 \\ 3 \end{bmatrix}$ so $W = \text{Span}\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 3 \end{bmatrix} \right\}$.
W is a subspace, since by Thm 1 in Section 4.1 the span of a set of vectors in a space V is a subspace of V.

(2) True or false (no working needed, just circle the answer):

(a) T : The set $M_{2\times 3}$ of 2×3 matrices with real entries is a vector space.

(b) F: $\mathbb{R}^2$ is a subspace of $\mathbb{R}^3$.

(c) T : If A is invertible, its columns form a basis for Col$A$.

(d) T : If A is invertible, Nul$A = \{\mathbf{0}\}$.

(e) T : Any nonempty subset of a basis is linearly independent.

(3) Consider the matrix

$$B = \begin{bmatrix} 2 & 4 & 2 & 13 & 2 \\ 1 & 2 & 0 & 4 & -2 \\ 2 & 4 & -1 & 8 & -2 \\ 1 & 2 & -1 & 3 & -2 \end{bmatrix}.$$

(a) Compute a basis for Nul$B$, which is a subspace of $\mathbb{R}^5$.

$$\begin{bmatrix} 2 & 4 & 2 & 13 & 2 \\ 1 & 2 & 0 & 4 & -2 \\ 2 & 4 & -1 & 8 & -2 \\ 1 & 2 & -1 & 3 & -2 \end{bmatrix} \xrightarrow[R_3 \leftarrow R_3 - 2R_2]{R_1 \leftarrow R_1 - 2R_2} \begin{bmatrix} 0 & 0 & 2 & 5 & 6 \\ 1 & 2 & 0 & 4 & -2 \\ 0 & 0 & -1 & 0 & 2 \\ 1 & 2 & -1 & 3 & -2 \end{bmatrix} \xrightarrow{R_2 \leftarrow R_2 - R_4} \begin{bmatrix} 0 & 0 & 2 & 5 & 6 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 2 \\ 1 & 2 & -1 & 3 & -2 \end{bmatrix}$$

$$\xrightarrow[R_2 \leftarrow R_2 + R_3]{R_1 \leftarrow R_1 + 2R_3} \begin{bmatrix} 0 & 0 & 0 & 5 & 10 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & -1 & 0 & 2 \\ 1 & 2 & -1 & 3 & -2 \end{bmatrix} \xrightarrow[R_4 \to R_4 - R_3]{R_1 \leftarrow R_1 - 5R_2} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & -1 & 0 & 2 \\ 1 & 2 & 0 & 3 & -4 \end{bmatrix} \xrightarrow[R_3 \leftrightarrow R_2]{R_1 \leftrightarrow R_4} \begin{bmatrix} 1 & 2 & 0 & 3 & -4 \\ 0 & 0 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow[R_1 \leftarrow R_1 - 3R_3]{R_2 \leftrightarrow -R_2} \begin{bmatrix} 1 & 2 & 0 & 0 & -10 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\text{Nul}A = x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 10 \\ 0 \\ 2 \\ -2 \\ 1 \end{bmatrix} \text{ and a basis is } \left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 10 \\ 0 \\ 2 \\ -2 \\ 1 \end{bmatrix} \right\}.$$

(b) Compute a basis for Col$B$, which is a subspace of $\mathbb{R}^4$.

Solution. The basis for Col$A$ is given by the pivot columns in the original matrix, so our basis is $\left\{ \begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} 13 \\ 4 \\ 8 \\ 3 \end{bmatrix} \right\}$.