$\boxed{\#1}$ (a) Notice that $\begin{bmatrix} 4 \\ 5 \\ 8 \end{bmatrix} = 4 \begin{bmatrix} 1 \\ 5/4 \\ 2 \end{bmatrix}$. So the columns of A are not linearly independent, so A is not invertible.

(b)

$$\left[\begin{array}{ccc|ccc} 2 & 0 & 6 & 1 & 0 & 0 \\ 6 & 1 & 6 & 0 & 1 & 0 \\ -10 & -1 & -27 & 0 & 0 & 1 \end{array}\right] \xrightarrow[5R_1+R_3 \to R_3]{-3R_1+R_2 \to R_2} \left[\begin{array}{ccc|ccc} 2 & 0 & 6 & 1 & 0 & 0 \\ 0 & 1 & -12 & -3 & 1 & 0 \\ 0 & -1 & 3 & 5 & 0 & 1 \end{array}\right]$$

$$\xrightarrow[R_2+R_3 \to R_3]{\frac{1}{2}R_1 \to R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 1/2 & 0 & 0 \\ 0 & 1 & -12 & -3 & 1 & 0 \\ 0 & 0 & -9 & 2 & 1 & 1 \end{array}\right]$$

$$\xrightarrow{\frac{1}{3}R_3 \to R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 1/2 & 0 & 0 \\ 0 & 1 & -12 & -3 & 1 & 0 \\ 0 & 0 & -3 & 2/3 & 1 & 1 \end{array}\right]$$

$$\xrightarrow[R_2-4R_3 \to R_2]{R_1+R_3 \to R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 7/6 & 1 & 1 \\ 0 & 1 & 0 & -17/3 & -3 & -4 \\ 0 & 0 & -3 & 2/3 & 1 & 1 \end{array}\right]$$

$$\xrightarrow{-\frac{1}{3}R_3 \to R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 7/6 & 1 & 1 \\ 0 & 1 & 0 & -17/3 & -3 & -4 \\ 0 & 0 & 1 & -2/9 & -1/3 & -1/3 \end{array}\right]$$

A reduces to I_3.

So A is invertible and

$$A^{-1} = \begin{bmatrix} 7/6 & 1 & 1 \\ -17/3 & -3 & -4 \\ -2/9 & -1/3 & -1/3 \end{bmatrix}$$

(c) $\begin{bmatrix} -4 & 6 \\ 6 & -9 \end{bmatrix} \xrightarrow{\frac{1}{2}R_1 \to R_1} \begin{bmatrix} -2 & 3 \\ 6 & -9 \end{bmatrix} \xrightarrow{3R_1+R_2 \to R_2} \underbrace{\begin{bmatrix} -2 & 3 \\ 0 & 0 \end{bmatrix}}_{\text{only 1 pivot position}}$

The matrix has 1 pivot position, but not 2, so the matrix is not invertible.

$\boxed{\#2}$

(a) T

(b) F

(c) T

(d) F

(e) F

$\boxed{\#3}$

(a) $\det A = 1\begin{vmatrix} 2 & 2/3 \\ 3/2 & -1/2 \end{vmatrix} - 1\begin{vmatrix} 2 & -1 \\ 3/2 & -1/2 \end{vmatrix} + 0\begin{vmatrix} 2 & -1 \\ 2 & 2/3 \end{vmatrix}$

$= 1\cdot(-1-1) - 1(-1+3/2) + 0$

$= -2 - \frac{1}{2} = \boxed{-\frac{5}{2}}$

(b) $\det B = 6\begin{vmatrix} -5 & 3 \\ -1/3 & 2/3 \end{vmatrix} - 0\begin{vmatrix} -2 & 4 \\ -1/3 & 2/3 \end{vmatrix} + 1\begin{vmatrix} -2 & 4 \\ -5 & 3 \end{vmatrix}$

$= 6\left(\frac{-10}{3} + 1\right) - 0 + (-6+20)$

$= -14 + 14 = \boxed{0}$

(c) (i) $\det(AB^T) = (\det A)(\det B^T)$

$= (\det A)(\det B)$

$= -\frac{5}{2} \cdot 0 = 0$ $\boxed{\text{So not invertible.}}$

(ii) $\det((AB)^T) = \det(AB) = (\det A)(\det B) = 0.$

So $\boxed{\text{not invertible.}}$

(iii) $\det((A^T)^3) = \det(A^T \cdot A^T \cdot A^T)$

$= \det(A^T) \cdot \det(A^T) \cdot \det(A^T)$

$= \det(A) \cdot \det(A) \cdot \det(A)$

$= \left(-\frac{5}{2}\right)^3 \neq 0$ so $\boxed{\text{invertible}}$