HW02 Solutions

#1

5 points total

(a) (1 point)

$$x_1\begin{bmatrix}1\\1\\-3\end{bmatrix}+x_2\begin{bmatrix}3\\4\\-7\end{bmatrix}+x_3\begin{bmatrix}-5\\-8\\9\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

(b)

$$\left[\begin{array}{ccc|c}1&3&-5&0\\1&4&-8&0\\-3&-7&9&0\end{array}\right]\xrightarrow[R_2-R_1\to R_2]{}\left[\begin{array}{ccc|c}1&3&-5&0\\0&1&-3&0\\-3&-7&9&0\end{array}\right]$$

3 points for justification

$$\xrightarrow[R_3+3R_1\to R_3]{}\left[\begin{array}{ccc|c}1&3&-5&0\\0&1&-3&0\\0&2&-6&0\end{array}\right]\xrightarrow[R_3-2R_2\to R_3]{}\left[\begin{array}{ccc|c}\boxed{1}&3&-5&0\\0&\boxed{1}&-3&0\\0&0&0&0\end{array}\right]$$

no pivot.

Since there is no pivot position in the third column, there is a free variable. (x_3).

Because there is a free variable, the system has a nontrivial solution. 1 point for answer.

#2

5 points total

(a) T

(b) T

(c) F

(d) T

(e) F

#3

5 points total

(b)

$$\left[\begin{array}{ccc|c} 2 & -6 & 8 & 0 \\ -4 & 7 & h & 0 \\ 1 & -3 & 4 & 0 \end{array}\right] \xrightarrow[\to R_1]{R_2+2R_1} \left[\begin{array}{ccc|c} 2 & -6 & 8 & 0 \\ 0 & -5 & h+16 & 0 \\ 1 & -3 & 4 & 0 \end{array}\right]$$

+1

$$\xrightarrow{\frac{1}{2}R_2 \to R_2} \left[\begin{array}{ccc|c} 1 & -3 & 4 & 0 \\ 0 & -5 & h+16 & 0 \\ 1 & -3 & 4 & 0 \end{array}\right] \xrightarrow[\to R_1]{R_3-R_1} \left[\begin{array}{ccc|c} 1 & -3 & 4 & 0 \\ 0 & -5 & h+16 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

no pivot in this column

infinitely many solutions

+1

$x_1\vec{u} + x_2\vec{v} + x_3\vec{w} = \vec{0}$ has infinitely many solutions.

+1 So they are dependent for all values of h.

(a)

$$\left[\begin{array}{cc|c} 2 & -6 & 0 \\ -4 & 7 & 0 \\ 1 & -3 & 0 \end{array}\right] \xrightarrow{\text{same steps as above}} \left[\begin{array}{cc|c} 1 & -3 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

+1

pivot positions in both columns.

So we have a unique solution — the trivial solution.

+1

So $\vec{u}$ and $\vec{v}$ are linearly independent.