Your name:

Instructor (please circle): Samantha Allen Angelica Babei

Math 22 Fall 2018 Homework 1, due Fri Sept 21 4:00 pm in homework boxes in front of Kemeny 108 *Please show your work, and check your answers. No credit is given for solutions without work or justification.*

(1) Given the system of equations, answer the following questions.

$$\begin{aligned} x_3 + 2x_4 &= 7 \\ -2x_1 - 8x_2 + x_3 + 6x_4 &= 17 \\ x_1 + 4x_2 + x_3 + 3x_4 &= 14 \end{aligned}$$

(a) Write the augmented matrix and transform it to reduced echelon form, showing all your steps:

$$\begin{bmatrix} 0 & 0 & 1 & 2 & 7 \\ -2 & -8 & 1 & 6 & 17 \\ 1 & 4 & 1 & 3 & 14 \end{bmatrix} \xrightarrow{R_2 \leftarrow R_2 + 2R_3} \begin{bmatrix} 0 & 0 & 1 & 2 & 7 \\ 0 & 0 & 3 & 12 & 45 \\ 1 & 4 & 1 & 3 & 14 \end{bmatrix} \xrightarrow{R_2 \leftarrow R_1 - \frac{1}{3}R_2} \begin{bmatrix} 0 & 0 & 0 & -2 & -8 \\ 0 & 0 & 3 & 12 & 45 \\ 1 & 4 & 1 & 3 & 14 \end{bmatrix}$$

$$\xrightarrow{R_1 \leftrightarrow R_3} \begin{bmatrix} 1 & 4 & 1 & 3 & 14 \\ 0 & 0 & 3 & 12 & 45 \\ 0 & 0 & 0 & -2 & -8 \end{bmatrix} \xrightarrow{R_2 \leftarrow \frac{1}{3}R_2} \begin{bmatrix} 1 & 4 & 1 & 3 & 14 \\ 0 & 0 & 1 & 4 & 15 \\ 0 & 0 & 0 & -2 & -8 \end{bmatrix}$$

$$\xrightarrow{R_3 \leftarrow -\frac{1}{2}R_3} \begin{bmatrix} 1 & 4 & 1 & 3 & 14 \\ 0 & 0 & 1 & 4 & 15 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_1 - R_2} \begin{bmatrix} 1 & 4 & 0 & -1 & -1 \\ 0 & 0 & 1 & 4 & 15 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix}$$

$$\xrightarrow{R_2 \leftarrow R_2 - 4R_3} \begin{bmatrix} 1 & 4 & 0 & -1 & -1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix} \xrightarrow{R_1 \leftarrow R_1 + R_3} \begin{bmatrix} 1 & 4 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix}$$

(b) Write the *general* solution to the linear system, if there is one.

$$\begin{aligned} x_1 &= 3 - 4x_2 \\ x_2 & \quad \text{free} \\ x_3 &= -1 \\ x_4 &= 4 \end{aligned}$$

(2) True or false

(a) F: Two matrices are row equivalent if they have the same number of rows.

(b) T : A consistent system has one or more solutions.

(c) F: If every column of an augmented matrix contains a pivot, then the corresponding system is consistent.

(d) T : A consistent system of 3 equations in 5 variables always has free variables.

(e) F: A system of 5 equations in 3 variables is never consistent.

(3) For which value(s) of the coefficient $\mathbf{a}$ does the linear system below have infinitely many solutions?

$$\begin{aligned} x_1 \qquad\quad + \mathbf{a}x_3 &= 3 \\ 3x_1 + 2x_2 + 3x_3 &= 6 \\ 2x_1 + 2x_2 + 5x_3 &= 3 \end{aligned}$$

Show the row operations that you performed, and explain why your value(s) for $\mathbf{a}$ lead(s) to infinitely many solutions.

Solution.

$$\begin{bmatrix} 1 & 0 & \mathbf{a} & 3 \\ 3 & 2 & 3 & 6 \\ 2 & 2 & 5 & 3 \end{bmatrix} \xrightarrow{R_2 \leftarrow R_2 - 3R_1} \begin{bmatrix} 1 & 0 & \mathbf{a} & 3 \\ 0 & 2 & 3 - 3\mathbf{a} & -3 \\ 2 & 2 & 5 & 3 \end{bmatrix} \xrightarrow{R_3 \leftarrow R_3 - 2R_1} \begin{bmatrix} 1 & 0 & \mathbf{a} & 3 \\ 0 & 2 & 3 - 3\mathbf{a} & -3 \\ 0 & 2 & 5 - 2\mathbf{a} & -3 \end{bmatrix}$$

$$\xrightarrow{R_3 \leftarrow R_3 - R_2} \begin{bmatrix} 1 & 0 & \mathbf{a} & 3 \\ 0 & 2 & 3 - 3\mathbf{a} & -3 \\ 0 & 0 & 2 + \mathbf{a} & 0 \end{bmatrix}$$

A system has infinitely many solutions when we have free variables. In this case, this happens if the third row has no pivots, i.e. when $\mathbf{a} = -2$.