Your name:

Instructor (please circle): Samantha Allen Angelica Babei

Math 22 Fall 2018 Homework 8, due Fri Nov 9 4:00 pm in homework boxes in front of Kemeny 108 *Please show your work, and check your answers. No credit is given for solutions without work or justification.*

(1) Let $\mathbf{v}_1 = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 4 \\ -3 \end{bmatrix}$. Note that $\mathbf{v}_1$ and $\mathbf{v}_2$ are orthogonal.

(a) Find a vector $\mathbf{v}_3$ such that the set $B = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is an orthogonal set.

(b) Normalize each vector in B to find an orthonormal basis B' for $\mathbb{R}^3$.

(c) Write $\mathbf{y} = \begin{bmatrix} 0 \\ -3 \\ 2 \end{bmatrix}$ as a linear combination of the vectors in B'.

(d) Find the distance from $\mathbf{y}$ to the subspace W of $\mathbb{R}^3$ spanned by $\mathbf{v}_1$ and $\mathbf{v}_2$.

(2) True or false (no working needed, just circle the answer):

(a) T / F: If A is a 6×5 matrix such that $\dim \mathrm{Col} A = 3$, then $\dim\left((\mathrm{Row} A)^{\perp}\right) = 2$.

(b) T / F: If $S = \{\mathbf{u}_1, \ldots, \mathbf{u}_n\}$ is an orthogonal set of vectors in $\mathbb{R}^n$, then S is a basis for $\mathbb{R}^n$.

(c) T / F: If U is a square matrix with orthonormal columns, then U is invertible.

(d) T / F: For any subspace W of $\mathbb{R}^n$, the only element which is in both W and $W^{\perp}$ is the zero vector.

(e) T / F: If two vectors $\mathbf{u}$ and $\mathbf{v}$ are orthgonal, then $||\mathbf{u} + \mathbf{v}|| < ||\mathbf{u}|| + ||\mathbf{v}||$.

(3) Consider the Markov chain given by transition matrix $P = \begin{bmatrix} 0 & 0.2 \\ 1 & 0.8 \end{bmatrix}$ and initial vector $\mathbf{x}_0 = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$.

(a) Show that P is a regular matrix.

(b) Find $\mathbf{x}_2$.

(c) Find the steady-state vector $\mathbf{q}$ for P.