

Your name:

Instructor (please circle):

Samantha Allen

Angelica Babei

Math 22 Fall 2018 Homework 3, due Fri Oct 5 4:00 pm in homework boxes in front of Kemeny 108 *Please show your work, and check your answers. No credit is given for solutions without work or justification.*

(1) Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^4$ be the function given by the formula

$$T(x_1, x_2, x_3) = (x_1 + 2x_2 - x_3, 3x_1 + 6x_2, x_1 + 2x_2, x_2 + 4x_3)$$

(a) T is a linear transformation. What is the standard matrix A of T ?

(b) Is the transformation T one-to-one? Justify your answer.

(c) Is the transformation T onto? Justify your answer.

(2) True or false (no working needed, just circle the answer):

(a) T / F: Every matrix transformation is a linear transformation.

(b) T / F: The function $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ given by the formula
$$T(x_1, x_2) = (x_1 - x_2, 5x_1 + 2x_2, x_2 - 5)$$
is a linear transformation.

(c) T / F: If $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is the linear transformation corresponding to counter-clockwise rotation of $3\pi/4$ (or 135°), then $T(\mathbf{x}) = A\mathbf{x}$ where

$$A = \begin{bmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix}.$$

(d) T / F: A linear transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^4$ is never onto.

(e) T / F: For any 3×3 matrices A and B , $AB = BA$.

(3) Let A, B, C be the following matrices:

$$A = \begin{bmatrix} 2 & 3 \\ -1 & 1 \\ 0 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & 1 \\ 0 & 3 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & -1 \\ -2 & 0 \end{bmatrix}$$

(a) Calculate AB, AC , and $AB + AC$.

(b) Calculate $B + C$ and then multiply A and $B + C$ to get $A(B + C)$.