Solving a linear system

$$\begin{cases} -2x_1 & +2x_2 & +10x_3 & = & 2 \\ -3x_1 & +x_2 & +9x_3 & = & 5 \\ 4x_1 & -2x_2 & -14x_3 & = & -6 \end{cases}$$

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$$\left\{\begin{array}{rrrcr} -2x_1 & +2x_2 & +10x_3 & = & 2 \\ -3x_1 & +x_2 & +9x_3 & = & 5 \\ 4x_1 & -2x_2 & -14x_3 & = & -6 \end{array}\right. \xrightarrow{R_1 \leftarrow \frac{1}{2} R_1} \left\{\begin{array}{rrrcr} x_1 & -x_2 & -5x_3 & = & -1 \\ -3x_1 & +x_2 & +9x_3 & = & 5 \\ 4x_1 & -2x_2 & -14x_3 & = & -6 \end{array}\right.$$

Solving a linear system

$$\begin{cases} -2x_1 & +2x_2 & +10x_3 & = & 2 \\ -3x_1 & +x_2 & +9x_3 & = & 5 \\ 4x_1 & -2x_2 & -14x_3 & = & -6 \end{cases} \xrightarrow{R_1 \leftarrow \frac{1}{2} R_1} \begin{cases} x_1 & -x_2 & -5x_3 & = & -1 \\ -3x_1 & +x_2 & +9x_3 & = & 5 \\ 4x_1 & -2x_2 & -14x_3 & = & -6 \end{cases}$$

$$\left(\begin{array}{rrr|r} -2 & 2 & 10 & 2 \\ -3 & 1 & 9 & 5 \\ 4 & -2 & -14 & -6 \end{array}\right) \xrightarrow{R_1 \leftarrow \frac{1}{2} R_1} \left(\begin{array}{rrr|r} 1 & -1 & -5 & -1 \\ -3 & 1 & 9 & 5 \\ 4 & -2 & -14 & -6 \end{array}\right)$$

Solving a linear system

$$\begin{cases} x_1 & -x_2 & -5x_3 & = & -1 \\ -3x_1 & +x_2 & +9x_3 & = & 5 \\ 4x_1 & -2x_2 & -14x_3 & = & -6 \end{cases}$$

Solving a linear system

$$\left\{\begin{array}{rrrcr} x_1 & -x_2 & -5x_3 & = & -1 \\ -3x_1 & +x_2 & +9x_3 & = & 5 \\ 4x_1 & -2x_2 & -14x_3 & = & -6 \end{array}\right. \xrightarrow{R_2 \leftarrow R_2 + 3R_1}$$

Solving a linear system

$$\left\{\begin{array}{rrrcr} x_1 & -x_2 & -5x_3 & = & -1 \\ -3x_1 & +x_2 & +9x_3 & = & 5 \\ 4x_1 & -2x_2 & -14x_3 & = & -6 \end{array}\right. \xrightarrow{R_2 \leftarrow R_2 + 3R_1} \left\{\begin{array}{rrrcr} x_1 & -x_2 & -5x_3 & = & -1 \\ & -2x_2 & -6x_3 & = & 2 \\ 4x_1 & -2x_2 & -14x_3 & = & -6 \end{array}\right.$$

Solving a linear system

$$\left\{\begin{array}{rrrcr} x_1 & -x_2 & -5x_3 & = & -1 \\ -3x_1 & +x_2 & +9x_3 & = & 5 \\ 4x_1 & -2x_2 & -14x_3 & = & -6 \end{array}\right. \xrightarrow{R_2 \leftarrow R_2 + 3R_1} \left\{\begin{array}{rrrcr} x_1 & -x_2 & -5x_3 & = & -1 \\ & -2x_2 & -6x_3 & = & 2 \\ 4x_1 & -2x_2 & -14x_3 & = & -6 \end{array}\right.$$

$$\left(\begin{array}{rrr|r} 1 & -1 & -5 & -1 \\ -3 & 1 & 9 & 5 \\ 4 & -2 & -14 & -6 \end{array}\right) \xrightarrow{R_2 \leftarrow R_2 + 3R_1} \left(\begin{array}{rrr|r} 1 & -1 & -5 & -1 \\ 0 & -2 & -6 & 2 \\ 4 & -2 & -14 & -6 \end{array}\right)$$

Solving a linear system

$$\begin{cases} x_1 & -x_2 & -5x_3 & = & -1 \\ & -2x_2 & -6x_3 & = & 2 \\ 4x_1 & -2x_2 & -14x_3 & = & -6 \end{cases} \xrightarrow{R_3 \leftarrow R_3 - 4R_1} \begin{cases} x_1 & -x_2 & -5x_3 & = & -1 \\ & -2x_2 & -6x_3 & = & 2 \\ & 2x_2 & +6x_3 & = & -2 \end{cases}$$

Solving a linear system

$$\begin{cases} x_1 & -x_2 & -5x_3 & = & -1 \\ & -2x_2 & -6x_3 & = & 2 \\ 4x_1 & -2x_2 & -14x_3 & = & -6 \end{cases} \xrightarrow{R_3 \leftarrow R_3 - 4R_1} \begin{cases} x_1 & -x_2 & -5x_3 & = & -1 \\ & -2x_2 & -6x_3 & = & 2 \\ & 2x_2 & +6x_3 & = & -2 \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & -1 & -5 & -1 \\ 0 & -2 & -6 & 2 \\ 4 & -2 & -14 & -6 \end{array}\right) \xrightarrow{R_3 \leftarrow R_3 - 4R_1} \left(\begin{array}{ccc|c} 1 & -1 & -5 & -1 \\ 0 & -2 & -6 & 2 \\ 0 & 2 & 6 & -2 \end{array}\right)$$

Solving a linear system

$$\begin{cases} x_1 & -x_2 & -5x_3 & = & -1 \\ & -2x_2 & -6x_3 & = & 2 \\ & 2x_2 & +6x_3 & = & -2 \end{cases} \xrightarrow{R_3 \leftarrow R_3 + R_2} \begin{cases} x_1 & -x_2 & -5x_3 & = & -1 \\ & -2x_2 & -6x_3 & = & 2 \\ & & 0 & = & 0 \end{cases}$$

Solving a linear system

$$\begin{cases} x_1 & -x_2 & -5x_3 & = & -1 \\ & -2x_2 & -6x_3 & = & 2 \\ & 2x_2 & +6x_3 & = & -2 \end{cases} \xrightarrow{R_3 \leftarrow R_3 + R_2} \begin{cases} x_1 & -x_2 & -5x_3 & = & -1 \\ & -2x_2 & -6x_3 & = & 2 \\ & & 0 & = & 0 \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & -1 & -5 & -1 \\ 0 & -2 & -6 & 2 \\ 0 & 2 & 6 & -2 \end{array}\right) \xrightarrow{R_3 \leftarrow R_3 + R_2} \left(\begin{array}{ccc|c} 1 & -1 & -5 & -1 \\ 0 & -2 & -6 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

Solving a linear system

$$\begin{cases} x_1 & -x_2 & -5x_3 & = & -1 \\ & -2x_2 & -6x_3 & = & 2 \end{cases} \xrightarrow{R_3 \leftarrow -\frac{1}{2}R_3} \begin{cases} x_1 & -x_2 & -5x_3 & = & -1 \\ & x_2 & +3x_3 & = & -1 \end{cases}$$

Solving a linear system

$$\begin{cases} x_1 & -x_2 & -5x_3 & = & -1 \\ & -2x_2 & -6x_3 & = & 2 \end{cases} \xrightarrow{R_3 \leftarrow -\frac{1}{2}R_3} \begin{cases} x_1 & -x_2 & -5x_3 & = & -1 \\ & x_2 & +3x_3 & = & -1 \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & -1 & -5 & -1 \\ 0 & -2 & -6 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right) \xrightarrow{R_3 \leftarrow -\frac{1}{2}R_3} \left(\begin{array}{ccc|c} 1 & -1 & -5 & -1 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

Solving a linear system

$$\begin{cases} x_1 - x_2 - 5x_3 = -1 \\ \quad\;\; x_2 + 3x_3 = -1 \end{cases} \xrightarrow{R_1 \leftarrow R_1 + R_2} \begin{cases} x_1 - x_2 - 5x_3 = -1 \\ \quad\;\; x_2 + 3x_3 = -1 \end{cases}$$

Solving a linear system

$$\begin{cases} x_1 - x_2 - 5x_3 = -1 \\ \quad\;\; x_2 + 3x_3 = -1 \end{cases} \xrightarrow{R_1 \leftarrow R_1 + R_2} \begin{cases} x_1 - x_2 - 5x_3 = -1 \\ \quad\;\; x_2 + 3x_3 = -1 \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & -1 & -5 & -1 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right) \xrightarrow{R_1 \leftarrow R_1 + R_2} \left(\begin{array}{ccc|c} 1 & 0 & -2 & -2 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

Solving a linear system

$$\left(\begin{array}{ccc|c} 1 & 0 & -2 & -2 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

Solving a linear system

$$\left(\begin{array}{ccc|c} 1 & 0 & -2 & -2 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right) \quad \rightsquigarrow \quad \begin{cases} x_1 -2x_3 = -2 \\ x_2 + 3x_3 = -1 \end{cases}$$

Solving a linear system

$$\left(\begin{array}{ccc|c} 1 & 0 & -2 & -2 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right) \quad \rightsquigarrow \quad \begin{cases} x_1 \qquad -2x_3 = -2 \\ \qquad x_2 + 3x_3 = -1 \end{cases}$$

$$\Longrightarrow \begin{cases} x_1 \quad = 2x_3 - 2 \\ x_2 = -3x_3 - 1 \end{cases}$$

$$\Longrightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

Solving a linear system

$$\left(\begin{array}{ccc|c} 1 & 0 & -2 & -2 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right) \quad \rightsquigarrow \quad \begin{cases} x_1 \qquad -2x_3 = -2 \\ \qquad x_2 + 3x_3 = -1 \end{cases}$$

$$\Longrightarrow \begin{cases} x_1 = 2x_3 - 2 \\ x_2 = -3x_3 - 1 \end{cases}$$

$$\Longrightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2t - 2 \\ -3t - 1 \\ t \end{pmatrix}$$

Solving a linear system

$$\left(\begin{array}{ccc|c} 1 & 0 & -2 & -2 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right) \quad \rightsquigarrow \quad \begin{cases} x_1 \qquad -2x_3 = -2 \\ \qquad x_2 + 3x_3 = -1 \end{cases}$$

$$\Longrightarrow \begin{cases} x_1 = 2x_3 - 2 \\ x_2 = -3x_3 - 1 \end{cases}$$

$$\Longrightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2t - 2 \\ -3t - 1 \\ t \end{pmatrix} = \begin{pmatrix} 2t \\ -3t \\ t \end{pmatrix} + \begin{pmatrix} -2 \\ -1 \\ 0 \end{pmatrix}$$

Solving a linear system

$$\left(\begin{array}{ccc|c} 1 & 0 & -2 & -2 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right) \quad \rightsquigarrow \quad \begin{cases} x_1 \qquad -2x_3 = -2 \\ \qquad x_2 + 3x_3 = -1 \end{cases}$$

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Solving a linear system

$$\left(\begin{array}{ccc|c} 1 & 0 & -2 & -2 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right) \quad \rightsquigarrow \quad \begin{cases} x_1 \qquad -2x_3 = -2 \\ \qquad x_2 + 3x_3 = -1 \end{cases}$$

$$\Longrightarrow \begin{cases} x_1 = 2x_3 - 2 \\ x_2 = -3x_3 - 1 \end{cases}$$

$$\Longrightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2t-2 \\ -3t-1 \\ t \end{pmatrix} = \begin{pmatrix} 2t \\ -3t \\ t \end{pmatrix} + \begin{pmatrix} -2 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} t + \begin{pmatrix} -2 \\ -1 \\ 0 \end{pmatrix}$$

(Parametric form)