Your name:

Instructor (please circle): Alex Barnett Naomi Tanabe

Math 22 Fall 2016, Midterm 2, Wed Oct 26

Please show your work. No credit is given for solutions without work or justification.

1. [8 points] Compute the determinants of the matrices in (a) and (b) (in each case there is a way that is quite quick).

(a) $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 2 & 0 & 3 \\ 1 & -7 & 2 & 5 \\ 4 & 9 & 3 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 5 \\ 3 & 5 & 4 \end{bmatrix}$

(c) Explain why if A is a 3×3 matrix, $\det A = \det A^T$.

2. [9 points] Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

(a) Find (and simplify) the characteristic polynomial for A.

(b) Find the eigenvalues of A with their multiplicities. For each, give a basis for its eigenspace.

(c) Evaluate $A^4 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

3. [9 points] Define the set of vectors $H = \left\{ \begin{bmatrix} a+b+2c \\ -b-c \\ 2a+b+3c \end{bmatrix} : \quad a, b, c \text{ real} \right\}$.

(a) Explain why H is a vector space (you may use results from class).

(b) Find a basis for H.

(c) Is $H = \mathbb{R}^3$?

(d) Each vector in H is a linear combination of the linearly independent standard basis vectors $\mathbf{e}_1$, $\mathbf{e}_2$ and $\mathbf{e}_3$. Are these vectors a basis for H, and why?

(e) For what p is H isomorphic to $\mathbb{R}^p$? (no explanation needed here)

4. [8 points]

(a) Is the set $V = \left\{ \begin{bmatrix} 2a+1 \\ a+1 \end{bmatrix} \ : \ a \text{ real} \right\}$ a vector space? Prove your answer.

(b) Let A be any matrix. Then is the set Nul A a vector space? Prove your answer.

(c) If all solutions to a homogeneous 4×5 linear system are multiples of one nontrivial vector, then must the linear system be consistent whatever constants are chosen for the right-hand side? Explain.

BONUS: Let A be a $m \times n$ matrix with Nul $A = \mathbb{R}^n$. What can you prove about A ?

5. [8 points]

(a) Give the definition of a set of vectors $\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ being a *basis* for a vector space V.

(b) Show that $\mathfrak{B} = \{t^2 + 1, t - 2, t + 3\}$ is a basis for $\mathbb{P}_2$.

(c) Let $\mathbf{v} = 8t^2 - 4t + 6$. Find its coordinate vector $[\mathbf{v}]_{\mathfrak{B}}$ relative to $\mathfrak{B}$ in part (b).

6. [8 points] In this question only, no working is needed; just circle T or F.

(a) T / F: Row reduction of a square matrix preserves its eigenvalues.

(b) T / F: If the vectors $\mathbf{v}_1, \ldots, \mathbf{v}_p$ span a vector space V, then $\dim V = p$.

(c) T / F: If A and B are row-equivalent, then rank $A =$ rank B.

(d) T / F: If A is an $n \times (n-1)$ matrix and rank $A = n-2$, then dim Nul $A = 2$.

(e) T / F: For sufficiently small positive ϵ the computer will report the rank of the matrix $\begin{bmatrix} 1 & 1 \\ 1 & 1+\epsilon \end{bmatrix}$ as one.

(f) T / F: $\mathbb{R}^6$ is a subspace of $\mathbb{R}^7$.

(g) T / F: The matrix $\begin{bmatrix} -7 & -5 \\ 10 & 5 \end{bmatrix}$ has no real eigenvalues.

(h) T / F: The subset of continuous functions on $[0,1]$ with $\int_0^1 f(t)dt = 0$ is a subspace of the set of continuous functions on $[0,1]$.