

ORTHOGONAL DIAGONALIZATION WORKSHEET

NOVEMBER 8, 2017

1. Let $A = \begin{pmatrix} 5 & 3 \\ 3 & 5 \end{pmatrix}$.

(a) Orthogonally diagonalize A .

(b) Construct a spectral decomposition for A .

2. Let A be a real symmetric matrix. Suppose that $\mathbf{x} \in \mathbb{C}^n$ is an eigenvector of A with corresponding eigenvalue λ . (Note that \mathbf{x} is a complex vector and λ is a complex number.)

(a) Express $\bar{\mathbf{x}}^T A \mathbf{x}$ in terms of λ , \mathbf{x} , and $\bar{\mathbf{x}}$.

(b) Express $\mathbf{x}^T \overline{A \mathbf{x}}$ in terms of λ , \mathbf{x} , and $\bar{\mathbf{x}}$.

(c) Show that $\overline{\bar{\mathbf{x}}^T A \mathbf{x}} = \bar{\mathbf{x}}^T A \mathbf{x}$.

(d) Show that $\bar{\lambda} = \lambda$ and conclude that λ is real.