ORTHOGONAL SETS WORKSHEET

NOVEMBER 1, 2017

Let

$$\mathbf{v}_1 = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} -1 \\ 4 \\ 1 \end{bmatrix}.$$

(a) Show that $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is an orthogonal basis for W.

(b) Let $\mathbf{y} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$. Without row-reducing, find the coordinates of $\mathbf{y}$ with respect to the basis $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$. That is, find $c_1, c_2, c_3 \in \mathbb{R}$ so that $\mathbf{y} = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3$.

(c) Compute the orthogonal projection of $\mathbf{y}$ onto $\mathbf{v}_2$.

(d) What is the shortest distance from $\mathbf{y}$ to $\text{span}\{\mathbf{v}_2\}$?