RANK-NULLITY WORKSHEET

OCTOBER 18, 2017

(1) Let

$$A = \begin{pmatrix} 1 & 2 & 1 & 0 \\ -1 & -2 & 2 & 3 \\ 0 & 0 & 5 & 5 \end{pmatrix}.$$

(a) Find a basis for $\mathrm{Col}(A)$. What is $\mathrm{rank}(A)$?

(b) Find a basis for $\mathrm{Nul}(A)$. What is $\mathrm{nullity}(A)$?

(c) Note that A is a 3×4 matrix and $\mathrm{rank}(A) + \mathrm{nullity}(A) = 4$. Can you explain why $\mathrm{rank}(B) + \mathrm{nullity}(B) = n$ for every $m \times n$ matrix B? (*Hint*: Think about $\mathrm{rank}(B)$ and $\mathrm{nullity}(B)$ in terms of pivots.)

(2) (a) With A as defined in the previous problem, find a basis of $\text{Row}(A)$.

(b) Note that $\dim(\text{Col}(A)) = \dim(\text{Row}(A))$. Do you think this equality holds for every matrix? Why or why not? (*Hint*: Think about pivots.)