CHEAT SHEET FOR WRITING PROOFS (MATH 54)

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LIST OF LOGICAL SYMBOLS

Feel free to use any of the following symbols in your homework. If you use a lot of symbols, start each sentence on a new line and leave lots of white space so it's easier to read.

Symbol	Meaning
$A \Rightarrow B$	"If A, then B." In other words, "A implies B." In other words, "If A is true, then B is true."
$A \Leftarrow B$	"If B, then A." In other words, "$B \Rightarrow A$."
$A \Leftrightarrow B$	"A if and only if B." In other words, "If A is true, then B is true. If A is false then B is false."
$\because$	"Because"
$\therefore$	"Therefore"
$\forall$	"For all"/"For every"
$\exists$	"There is at least one"
$\exists!$	"There is only one"
s.t.	"Such that"/"So that"
$\in$	"Is in" or "in," depending on context.
$\notin$	"Is not in" or "not in," depending on context.
$A := B$	"Let A equal B." or "A, which has been defined to be equal to B," depending on context.
$A \stackrel{\text{def}}{=} B$	Same as "$A := B$."

Examples:

- "$\forall x \in \mathbb{R}, \exists y \in \mathbb{R}$ s.t. $y > x$." means "For any real number, there is a bigger real number."
- "$\forall \mathbf{v}_1, \ldots, \mathbf{v}_p \in \mathbb{R}^n, \mathbf{0} \in \text{Span}\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$" means "The zero vector in $\mathbb{R}^n$ is a linear combination of any list of vectors in $\mathbb{R}^n$."

LIST OF SYMBOLS RELATING TO FUNCTIONS

$F \colon X \to Y$	"F is a function whose domain is X and whose codomain is Y"
$F \colon x \mapsto y$	"F is a function, x is in its domain, and $y = F(x)$." In other words, "F sends x to y."
$F(x) \equiv 1$	"$F(x)$ is equal to 1 for all x." (Saying "$F(x) = 1$" might mean that x is a number for which $F(x) = 1$, but there may be some other number y for which $F(y) \neq 1$.)

HOW TO PROVE "IF A THEN B" STATEMENTS

- Directly, i.e. working forward: Assume A is true, and then show B is true.
- Contrapositive, i.e. working backward: Assume B is false, then show A is false.
- Contradiction, i.e. work forward and backward at the same time: Assume A is true and B is false, then show that these two assumptions together break logic.
- Make sure you prove "If A then B" instead of "If B then A."

HOW TO PROVE "THERE EXIST" STATEMENTS

- Give a concrete example. Numbers are usually much better examples than formulas.
- Appeal to a theorem that guarantees the thing you want exists, even if that theorem doesn't give a concrete description of it.
- Combine the previous two strategies: Appeal to a theorem to show something exists, then use it to build the example you want.

GUIDELINES FOR WRITING PROOFS

These are guidelines, not rules. But you should still follow them unless you have a very good reason not to.

- Don't use introduce new letters, symbols, terminology, or assumptions without explaining what they are.
- Don't use the same letter or symbol to refer to two different things.
- Use complete and grammatically correct sentences (even when they include symbols).
- When proving statements, start with *exactly* what the problem gives you, and stop when you end up with *exactly* what you wanted to prove.
- Do not say that there is an example or a counterexample to a statement without proving it!