Barnett
10/3/16

— SOLUTIONS —

Your name:

Math 22 Fall 2016, mini-quiz 1, Mon Oct 3

Please show your work. No credit is given for solutions without work or justification.

2 pts. (1) Define what it means for a set of vectors $\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ to be linearly independent:

$\{\vec{v}_1, \ldots \vec{v}_n\}$ is L.I. if

whenever $x_1\vec{v}_1 + \cdots + x_n\vec{v}_n = \vec{0}$,

this implies $x_1 = x_2 = \cdots = x_n = 0$,

ie the weights are trivial.

3 pts (2) Is the matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 0 \\ 2 & 4 & 1 \end{bmatrix}$ invertible? Explain what result you used to deduce this.

Row reduce:

EF:

$$A \sim \begin{bmatrix} 1 & 2 & 0 \\ 2 & 4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \sim \begin{bmatrix} \boxed{1} & 2 & 0 \\ 0 & \boxed{1} & 0 \\ 0 & 0 & \boxed{1} \end{bmatrix}$$

all three pivots

Thus A is reducible to I_3

Thus (by Thm. 7 in §2.2, the "refinv" theorem),

A is invertible.

Note we didn't need to compute A^{-1}! We know it exists.

(3) Let $T : \mathbb{R}^3 \to \mathbb{R}^4$ be a linear transformation.

3 pts (a) Could T be onto? Prove your answer

No, since all lin. xforms have a standard matrix, so $T(\vec{x}) = A\vec{x}$ for a 4×3 matrix A.

A can have at most 3 pivots, thus cannot have one in every row, so there are some $\vec{b}$ in $\mathbb{R}^4$ for which $A\vec{x} = \vec{b}$ is inconsistent.

2 pts. (b) Could T be one-to-one? Prove your answer

Yes, it could be.

Simplest proof is by example: $T(\vec{x}) = A\vec{x}$ for

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$ is one-to-one because

there is a pivot in every column, so $A\vec{x} = \vec{b}$ either has zero or one solutions $\vec{x}$ for each $\vec{b}$ in $\mathbb{R}^4$. This is the definition of one-to-one.