9/13/18
Barnett

MATH 22 WORKSHEET : proofs

You will analyze exhaustively the "1×1 linear system" $ax=b$, with a,b numbers. ← x is a single unknown.

For each claim, if true prove it, or if false find a counterexample or prove false.
After each, swap with your neighbors & discuss whether yours is watertight:

i) Claim: if $b=0$, $ax=b$ is consistent for any a. T / F

ii) Let a,b be any real numbers. Then $ax=b$ has a solution. T / F

iii) If $a\neq 0$ then $ax=b$ is always consistent and unique. T / F
← ie, for any b. ← don't forget! [Hint: let x & y be two solutions...]

iv) There is some choice of a and b such that $ax=b$ has exactly 2 solutions. [Hint: let x,y be those solutions, with $x\neq y$] T / F

v) If $b=0$ then $ax=b$ is always unique. T / F

~ SOLUTIONS ~

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For each claim, if true prove it, or if false find a counterexample or prove false. After each, swap with your neighbors & discuss whether yours is watertight:

i) Claim: if $b=0$, $ax=b$ is consistent for any a. T / F

Proof by example: $x=0$ solves $ax=0$ no matter what a is.
$\Rightarrow$ at least one solution. $\square$

ii) Let a,b be any real numbers. Then $ax=b$ has a solution. T / F

Counterexample: $a=0$, $b=1$. Ie, $0.x = 1$ has no solution. $\square$

iii) If $a \neq 0$ then $ax=b$ is always consistent and unique. T / F
ie, for any b.

Proof: $x = b/a$ exists since $a \neq 0$, and solves $ax=b$. $\Rightarrow$ consistent. For uniqueness, let y be another solution. Subtract $ax=b$ & $ay=b$ $\Rightarrow$ $a(x-y)=0$. $\Rightarrow$ $y=x$, unique. $\square$

iv) There is some choice of a and b such that $ax=b$ has exactly 2 solutions. [Hint: let x, y be those solutions, with $x \neq y$] T / F

Suppose there are 2 solutions x, y, $x \neq y$. Then $ax=b$ & $ay=b$. Subtract to get $a(x-y)=0$. So $a=0$. Since consistent, $b=0$. But then any x is a soln.

v) If $b=0$ then $ax=b$ is always unique. T / F

Counterexample: $a=0$ Then $0x=0$ is not unique. (easy).

Notes: can also do iv) by considering case $a=0$, $a \neq 0$ separately