MATH 22 WORKSHEET: Parametric vector form of solutions. 6/30/06

A) Let's find general solution to $A\vec{x} = \vec{0}$ ← homogeneous

for $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

Write augmented matrix & reduce to REF:

check: in reduced EF?

ans: $\begin{bmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{bmatrix}$

Using pivots & free vars write general soln:

$x_1 =$

$x_2 =$

$x_3 =$

if free, leave as itself (e.g. $x_1 = x_1$)

Now write as vectors: $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{x} = \alpha \vec{v}$

free parameter   const. vector

What is α?   What is $\vec{v} = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \end{bmatrix}$?

Would $A\vec{x} = \vec{b}$ be consistent for any $\vec{b}$? (hint: pivots).

B) Now with same A matrix, solve $A\vec{x} = \vec{b}$ with $\vec{b} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Reuse your above to get aug matrix in REF:

$\sim \begin{bmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{bmatrix}$

Gen. soln:

$x_1 =$

$x_2 =$

$x_3 =$

write as $\vec{x} = \vec{p} + \alpha \vec{v}$

const. vectors

What are $\vec{p} = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \end{bmatrix}$? $\vec{v} = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \end{bmatrix}$?

So, How is inhomogeneous solution set related to homogeneous?

MATH 22 WORKSHEET: Parametric vector form of solutions. 6/30/06 Barnett

~ SOLUTIONS ~

A) Let's find general solution to $A\vec{x} = \vec{0}$ ← homogeneous.

for $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

Write augmented matrix & reduce to REF:

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 4 & 5 & 6 & 0 \end{array}\right] \underset{R_2 \leftarrow R_2 - 4R_1}{\sim} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -3 & -6 & 0 \end{bmatrix} \underset{R_2 \leftarrow -\frac{1}{3}R_2}{\sim} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & 0 \end{bmatrix}$$

$\sim \; R_1 \leftarrow R_1 - 2R_2$

check: is reduced EF?

ans: $\begin{bmatrix} \boxed{1} & 0 & -1 & 0 \\ 0 & \boxed{1} & 2 & 0 \end{bmatrix}$

pivots, free

Using pivots & free vars write general soln:

$$\left.\begin{aligned} x_1 &= 0 + x_3 \\ x_2 &= 0 - 2x_3 \\ x_3 &= \text{free} = x_3 \end{aligned}\right\} \quad \text{Now write as vectors: } \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{x} = \alpha \vec{v}$$

if free, leave as itself. (eg. $x_1 = x_1$)

free parameter; const. vector

What is α? What is $\vec{v} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$?

Would $A\vec{x} = \vec{b}$ be consistent for any $\vec{b}$? (hint: pivots). → Yes since pivot in every row.

B) Now with same A matrix, solve $A\vec{x} = \vec{b}$ with $\vec{b} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Reuse your above to get aug matrix in REF:

$$\text{RHS} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 - 4R_1} \begin{bmatrix} 1 \\ -3 \end{bmatrix} \xrightarrow{R_2 \leftarrow -\frac{1}{3}R_2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \underset{\sim}{R_1 \leftarrow R_1 - 2R_2} \begin{bmatrix} -1 \\ 1 \end{bmatrix} \sim \overbrace{\left[\begin{array}{ccc|c} 1 & 0 & -1 & -1 \\ \cdot & 1 & 2 & 1 \end{array}\right]}^{\text{as before}}$$

insert

Gen. soln:

$$\left.\begin{aligned} x_1 &= -1 + 1x_3 \\ x_2 &= 1 - 2x_3 \\ x_3 &= 0 + x_3 \end{aligned}\right\} \quad \text{write as } \vec{x} = \vec{p} + \alpha\vec{v}$$

const. vectors

What are $\vec{p} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$? $\vec{v} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$?

So. How is inhomogeneous solution related to [illegible]