MATH 22 WORKSHEET: Matrix-vector product

6/28/16
Barnett

Find $A\vec{x}$ for $A = \begin{bmatrix} 5 & 1 \\ 0 & 1 \\ -1 & 1 \end{bmatrix}$, $\vec{x} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ two ways:

1) $A\vec{x} = x_1 \vec{a}_1 + x_2 \vec{a}_2 = \bigcirc \begin{bmatrix} \; \\ \; \end{bmatrix} + \bigcirc \begin{bmatrix} \; \\ \; \end{bmatrix} = \; ?$

linear combination of columns of A

2) Consider 2 vectors: $\begin{bmatrix} 5 & 1 \end{bmatrix}$ ⟵ what part of A is this?

$\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \vec{x}$

Take their 'dot' (inner) product:

Which entry of the answer $A\vec{x}$ does it give?

Find similar way to get other two entries of $A\vec{x}$:

Write the general rule for j^{th} entry in answer:

Practise: compute these products (if they exist – or explain why not):

$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \end{bmatrix} =$

$\begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} =$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} =$

$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} =$

$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} =$

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~~> SOLUTIONS <~~

Find $A\vec{x}$ for $A = \begin{bmatrix} 5 & 1 \\ 0 & 1 \\ -1 & 1 \end{bmatrix}$, $\vec{x} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ two ways:

1) $A\vec{x} = x_1 \vec{a}_1 + x_2 \vec{a}_2 = 2 \begin{bmatrix} 5 \\ 0 \\ -1 \end{bmatrix} + (-1) \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 9 \\ -1 \\ -3 \end{bmatrix}$?

linear combination of columns of A

2) Consider 2 vectors: $[5 \quad 1]$ ← what part of A is this?

$\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \vec{x}$

Take their 'dot' (inner) product: $2(5) + (-1)1 = 9$ Which entry of the answer $A\vec{x}$ does it give? 1st entry.

Find similar way to get other two entries of $A\vec{x}$:

$[0 \quad 1] \begin{bmatrix} 2 \\ -1 \end{bmatrix} = -1$, $[-1 \quad 1] \begin{bmatrix} 2 \\ -1 \end{bmatrix} = -3$.

Write the general rule for j^{th} entry in answer:

$(A\vec{x})_j$ = j row of A dotted into $\vec{x}$

Practise : compute these products (if they exist - or explain why not):

$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 17 \\ 39 \end{bmatrix}$ → dot = 17. Notice dot is fastest way mentally.

$\begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} =$ cannot do ! must be equal lengths.

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ ← the 'identity' matrix (3x3 version)

$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} z \\ x \\ y \end{bmatrix}$ ↖ permutation matrix

$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x\cos\theta - y\sin\theta \\ x\sin\theta + y\cos\theta \end{bmatrix}$ ← rotated the point :