

Let $\vec{a}_1 = \begin{bmatrix} 0 \\ 2 \\ 5 \end{bmatrix}$ $\vec{a}_2 = \begin{bmatrix} 1 \\ -3 \\ -8 \end{bmatrix}$ $\vec{a}_3 = \begin{bmatrix} -4 \\ 2 \\ h \end{bmatrix}$
← h is some number.

We want to know: Is $\vec{b} = \begin{bmatrix} 1 \\ 1 \\ k \end{bmatrix}$ in $\text{Span}\{\vec{a}_1, \vec{a}_2, \vec{a}_3\}$?
← k is some number.

Row reduce the augmented matrix to Echelon Form :

- treat h & k as you would usual numbers
- scale a row to remove fractions.

What set of h, k makes linear system

- a) consistent & unique?
- b) consistent but not unique?

Let $\vec{a}_1 = \begin{bmatrix} 0 \\ 2 \\ 5 \end{bmatrix}$

$\vec{a}_2 = \begin{bmatrix} 1 \\ -3 \\ -8 \end{bmatrix}$

$\vec{a}_3 = \begin{bmatrix} -4 \\ 2 \\ h \end{bmatrix}$

h is some number.

We want to know: Is $\vec{b} = \begin{bmatrix} 1 \\ 1 \\ k \end{bmatrix}$ in $\text{Span}\{\vec{a}_1, \vec{a}_2, \vec{a}_3\}$?
 k is some number.

Row reduce the augmented matrix to Echelon Form :

- treat h & k as you would usual numbers
- scale a row to remove fractions.

$$\left[\begin{array}{cccc|c} 0 & 1 & -4 & 1 & 1 \\ 2 & -3 & 2 & 1 & 1 \\ 5 & -8 & h & k & k \end{array} \right] \xrightarrow{R_2 \leftrightarrow R_1} \left[\begin{array}{cccc|c} 1 & -3/2 & 1 & 1/2 & 1/2 \\ 0 & 1 & -4 & 1 & 1 \\ 5 & -8 & h & k & k \end{array} \right]$$

$R_3 \rightarrow R_3 - 5R_1$

$R_3 \rightarrow R_3 + \frac{1}{2}R_2$

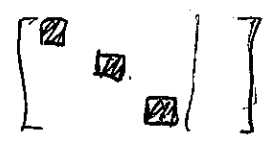
$$\sim \left[\begin{array}{cccc|c} 1 & -3/2 & 1 & 1/2 & 1/2 \\ 0 & 1 & -4 & 1 & 1 \\ 0 & -1/2 & h-5 & k-5/2 & k-5/2 \end{array} \right]$$

$$\sim \left[\begin{array}{cccc|c} 1 & -3/2 & 1 & 1/2 & 1/2 \\ 0 & 1 & -4 & 1 & 1 \\ 0 & 0 & h-7 & k-2 & k-2 \end{array} \right]$$

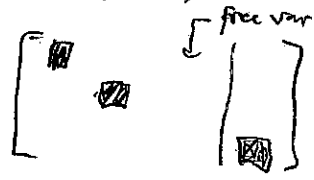
Echelon Form.

↑ actually, not important.

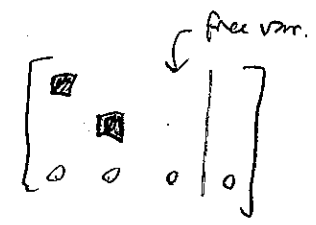
Examine pivot positions (depending on h, k) :



$h \neq 7$, any k.
consistent, unique



$h = 7$, $k \neq 2$
inconsistent.



$h = 7$, $k = 2$
consistent, non-unique.

What set of h, k makes linear system

a) consistent & unique? $h \neq 7$, $k = \text{anything}$

b) consistent but not unique? $h = 7$, $k = 2$

It's all about the pivots