Your name:

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Math 22 Fall 2016, Midterm 1, Mon Oct 10

Please show your work. No credit is given for solutions without work or justification.

1. [8 points] Consider the linear system $A\mathbf{x} = \mathbf{b}$ where

$$A = \begin{bmatrix} 0 & -1 & -1 \\ 2 & 4 & 0 \\ 1 & 1 & -1 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$$

   (a) Is the system consistent? If not, explain why. If consistent, write the general solution in *parametric vector form*:

   (b) Express the solution set to the corresponding *homogeneous* system $A\mathbf{x} = \mathbf{0}$ in the form of a span of one or more vectors:

2. [8 points] Consider the set of three vectors $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 8 \\ 7 \\ h \end{bmatrix}$.

(a) With $h = 3$, is the set linearly independent, and why? If not, give a dependence relation between $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$.

(b) For what value(s) of h, if any, does the set span $\mathbb{R}^3$?

3. [9 points]

(a) Give the definition of a square matrix A being invertible. [Note: do not give more than simply the definition.]

(b) Find the inverse of $A = \begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$, if it exists, or prove that it does not exist:

(c) Write a linear system involving A whose solution gives the 2nd column of A^{-1}:

4. [8 points]

(a) Give the definition of a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ being one-to-one. [Note: do not give more than simply the definition.]

(b) Let a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ rotate points (x, y, z) by $-\pi/2$ about the z-axis (ie, $90°$ counterclockwise about the origin when viewed from above) then discard the z coordinate (ie, output only x and y). Write the standard matrix for T:

(c) Is T from part (b) one-to-one? Why?

(d) Is T from part (b) onto? Why?

5. [7 points]

(a) Let A and B be square matrices, and let AB be invertible. Prove, from only this information, that B is invertible. [You are allowed to use any of the theorems from class, but of course you must state when you use them.]

(b) Let the 1st component h of a vector be the housed population, and the 2nd component s be the population living on the streets (homeless). Each year, 1% of those with houses become homeless. Also each year, half of the homeless find housing while the other half stays on the streets. Write the migration matrix which updates the population vector from year k to year $k+1$:

BONUS: Say the total population is 5.1 million. Find the *equilibrium* population vector, ie one that does not change from year to year:

6. [10 points] In this question only, no working is needed; just circle T or F.

(a) T / F: A linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ is onto if for each $\mathbf{x}$ in $\mathbb{R}^n$, there is a $\mathbf{b}$ in $\mathbb{R}^m$ such that $T(\mathbf{x}) = \mathbf{b}$.

(b) T / F: If A and B are invertible, then $((AB)^{-1})^T = (A^T)^{-1}(B^T)^{-1}$.

(c) T / F: The solution set to a linear system can be changed by elementary row operations.

(d) T / F: A linear transformation T is onto if every column of its standard matrix is a pivot column.

(e) T / F: Any set S of vectors containing the zero vector is linearly dependent.

(f) T / F: For a matrix equation $A\boldsymbol{x} = \mathbf{0}$ to be consistent, the augmented matrix $[A \,|\, \mathbf{0}]$ must have a pivot in every row.

(g) T / F: If any set of vectors $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3, \boldsymbol{v}_4\}$ is linearly dependent, then $\boldsymbol{v}_4$ can be written as a linear combination of $\boldsymbol{v}_1, \boldsymbol{v}_2$, and $\boldsymbol{v}_3$.

(h) T / F: A linear transformation T that maps $\mathbb{R}^n$ onto $\mathbb{R}^n$ is a one-to-one map.

(i) T / F: If A is invertible, then $A\boldsymbol{x} = \mathbf{0}$ has a nontrivial solution.

(j) T / F: A map that takes the zero element to some other element cannot be a linear transformation.